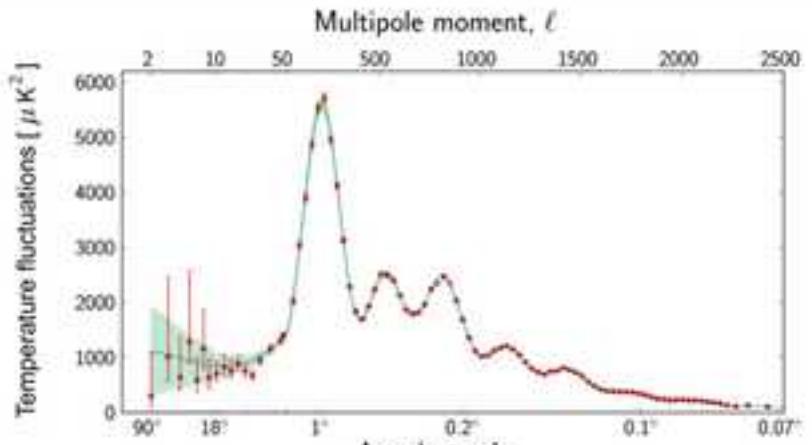
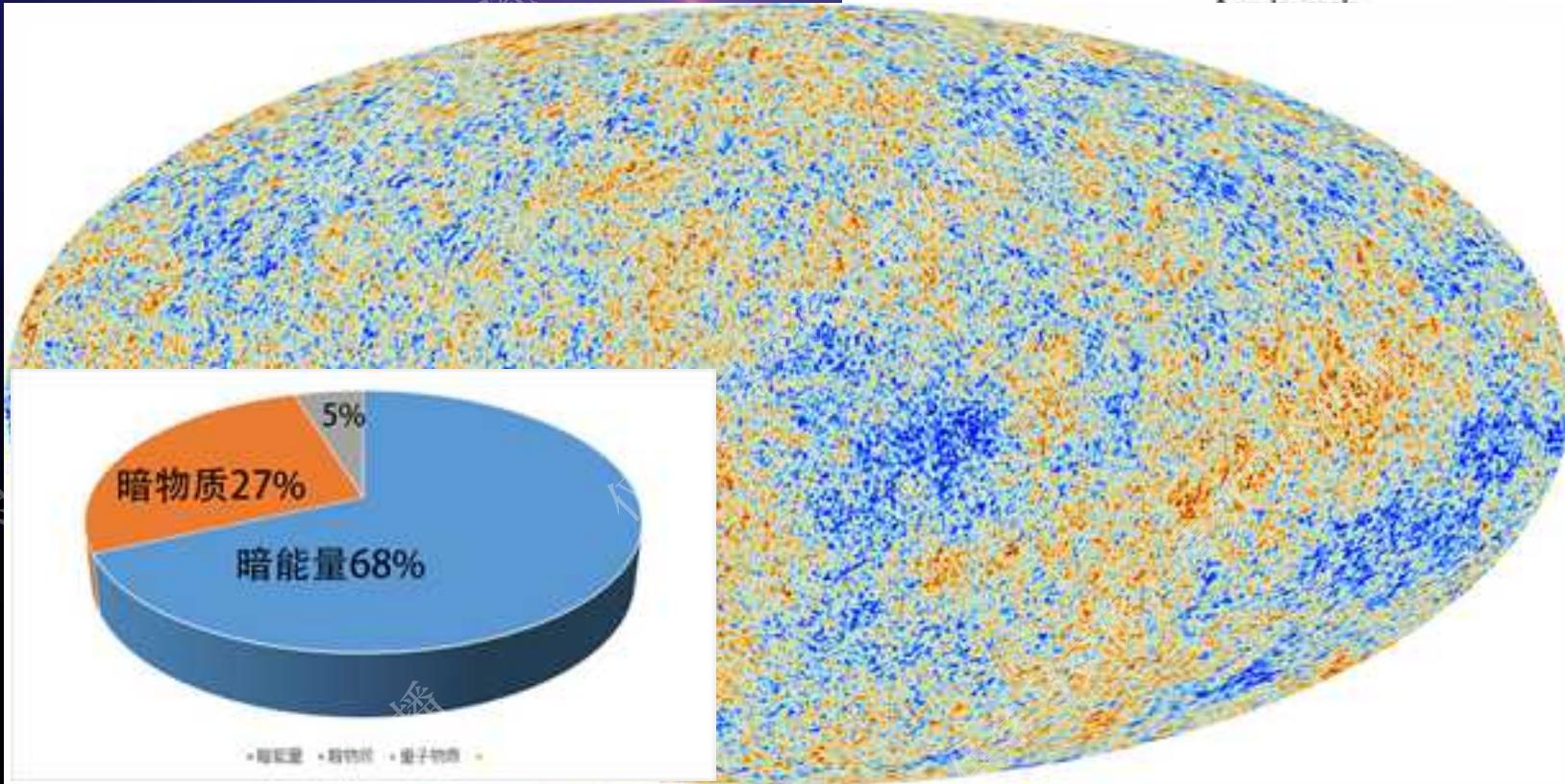


# LCDM小尺度挑战与暗物质属性

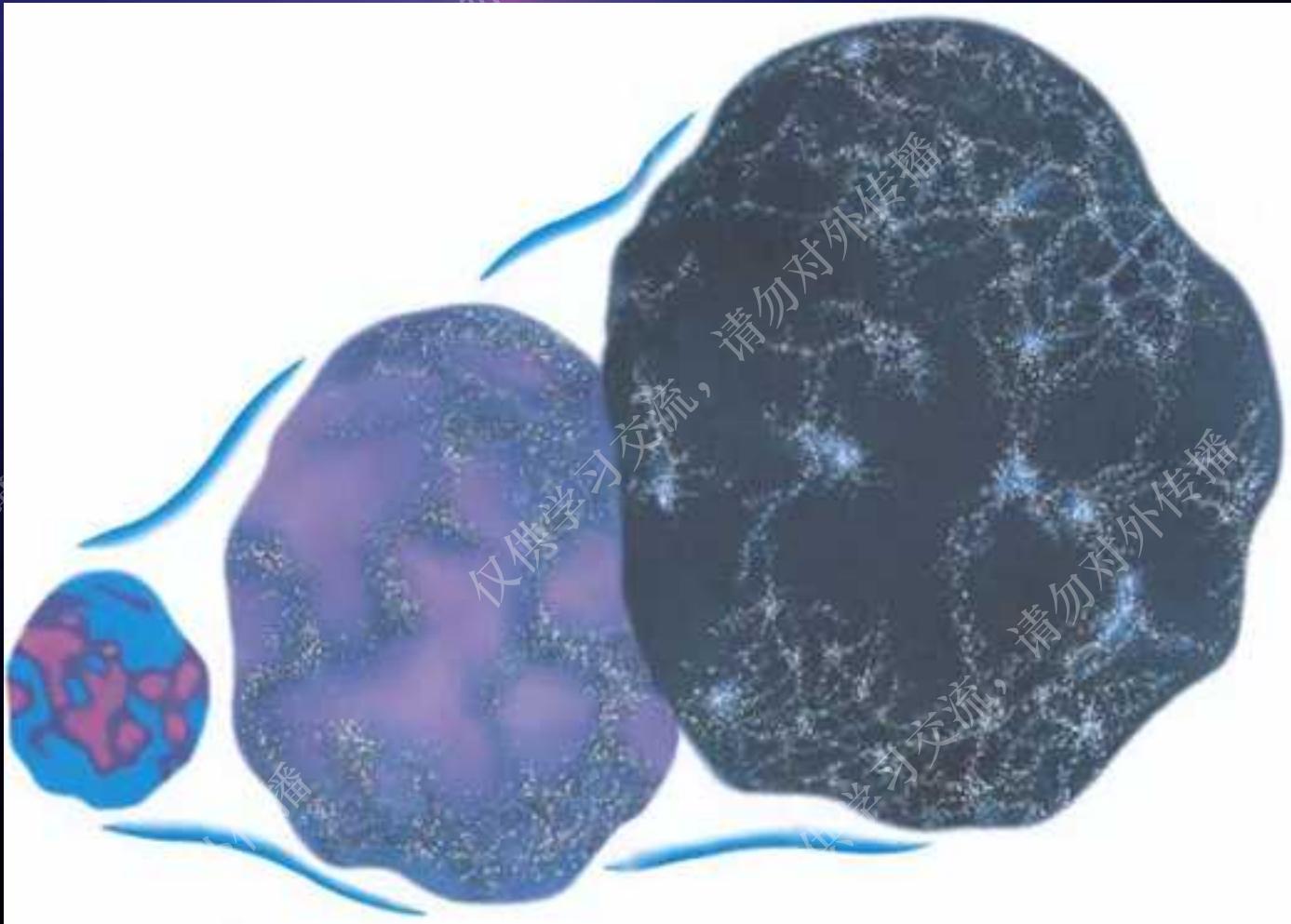
高亮  
计算宇宙学团组



# THE MICROWAVE SKY



These tiny fluctuations have evolved  
into clusters of galaxies today



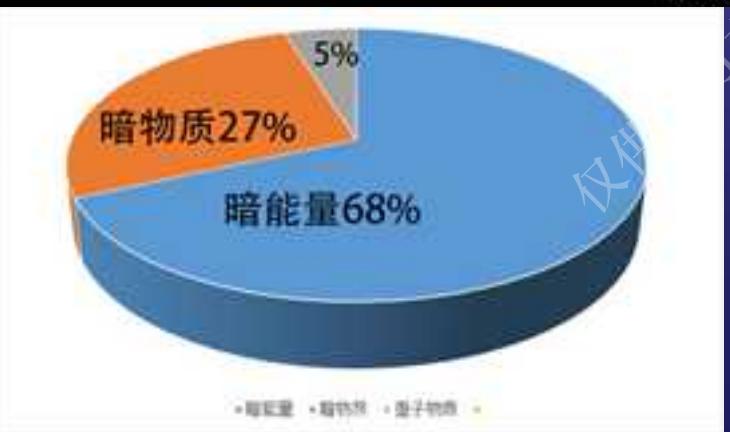
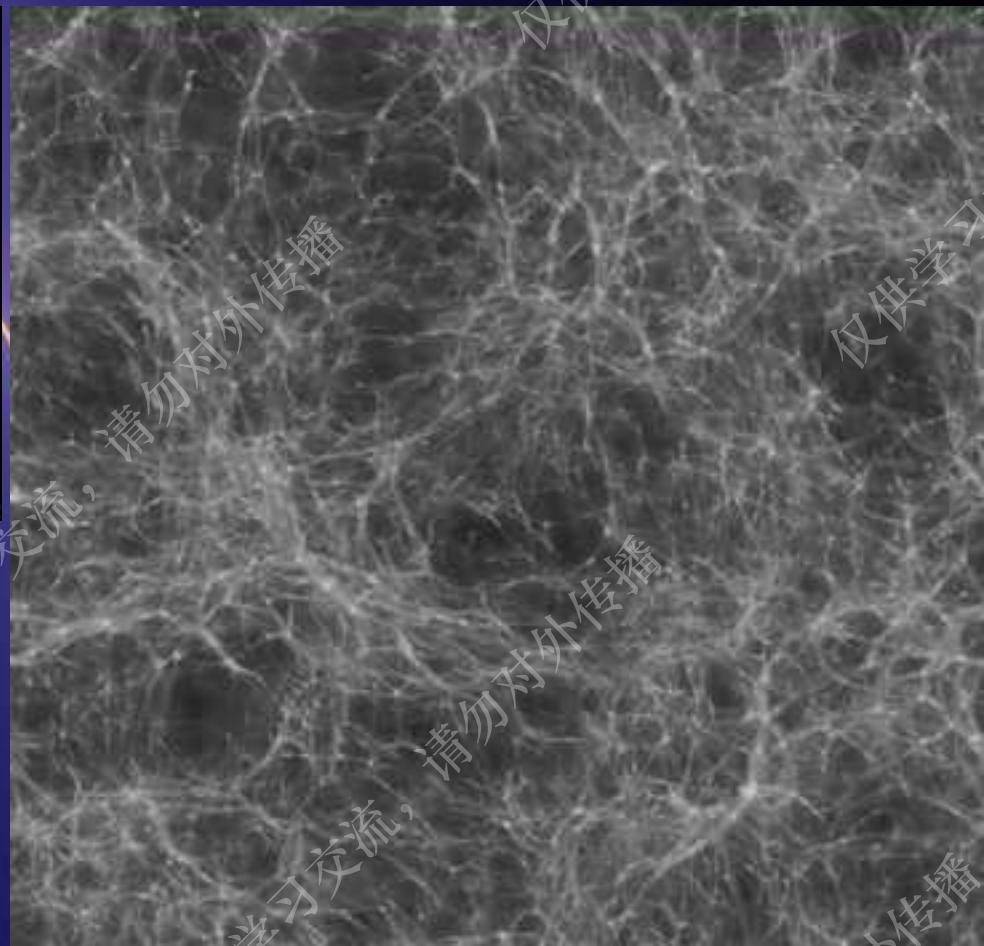
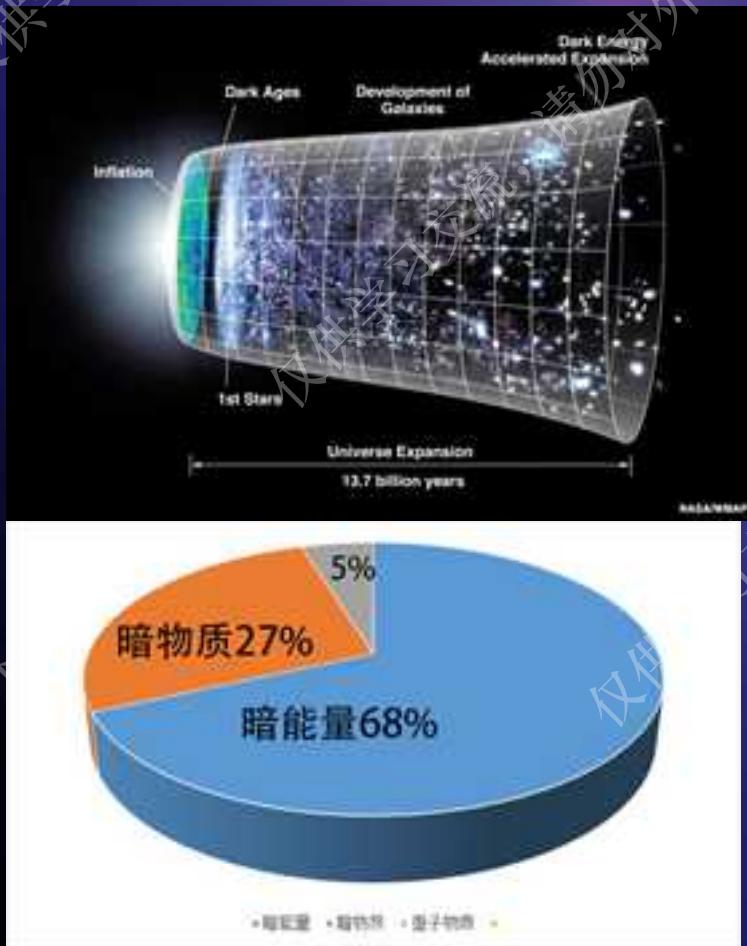
# 宇宙学结构形成物理—very simple!

- 引力主导宇宙所有结构形成
- 流体力学、热力学主导恒星/星系形成
- 微观物理（辐射、核反应）主导重元素的形成以及

产生光

**物理过程高度非线性，理论理解方面只能依靠数值模拟！**

# 用计算机重演宇宙形成演化历史



解释、指导天文观测；验证、完善理论

# 研究内容

## 宇宙物质组成成分：

- 27% 暗物质
- 5% 重子物质（气体、恒星）
- 68% 暗能量 → 宇宙加速膨胀

## 遵循的基本物理原理

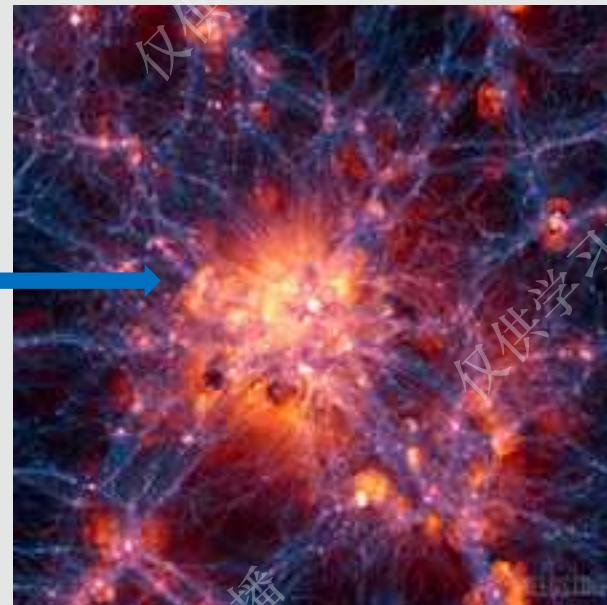
1) 暗物质、气体、恒星：引力（泊松方程）

2) 气体（理想气体方程）

$$\ddot{\mathbf{x}}_i = -\nabla_i \Phi(\mathbf{x}_i)$$

$$\Phi(\mathbf{x}) = -G \sum_{j=1}^N \frac{m_j}{[(\mathbf{x} - \mathbf{x}_j)^2 + \epsilon^2]}$$

3) 复杂天体物理过程（微观物理）



### 连续性方程

$$\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{v} = 0$$

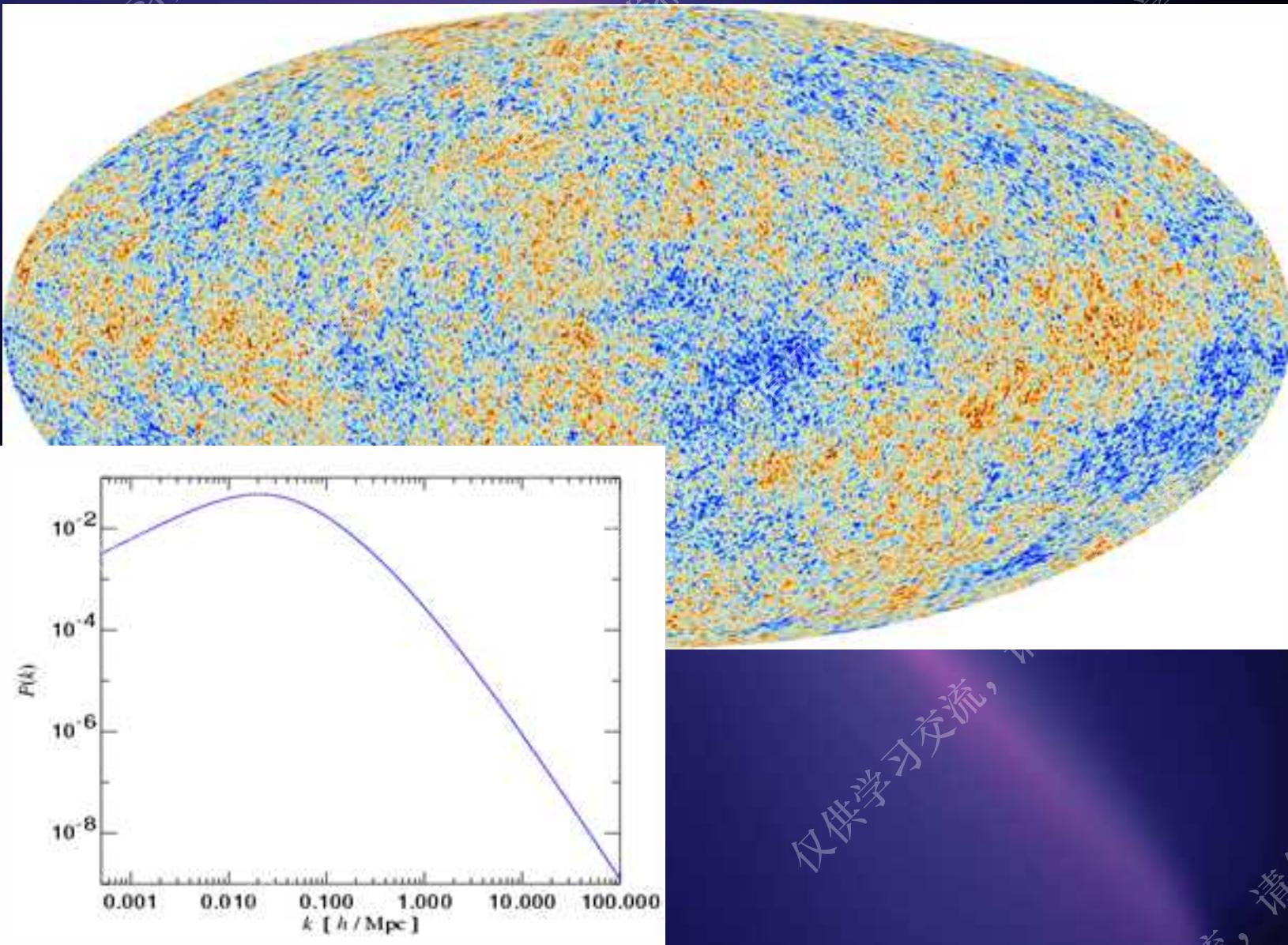
### 动量方程

$$\frac{d\mathbf{v}}{dt} = -\frac{\nabla P}{\rho} - \nabla \Phi$$

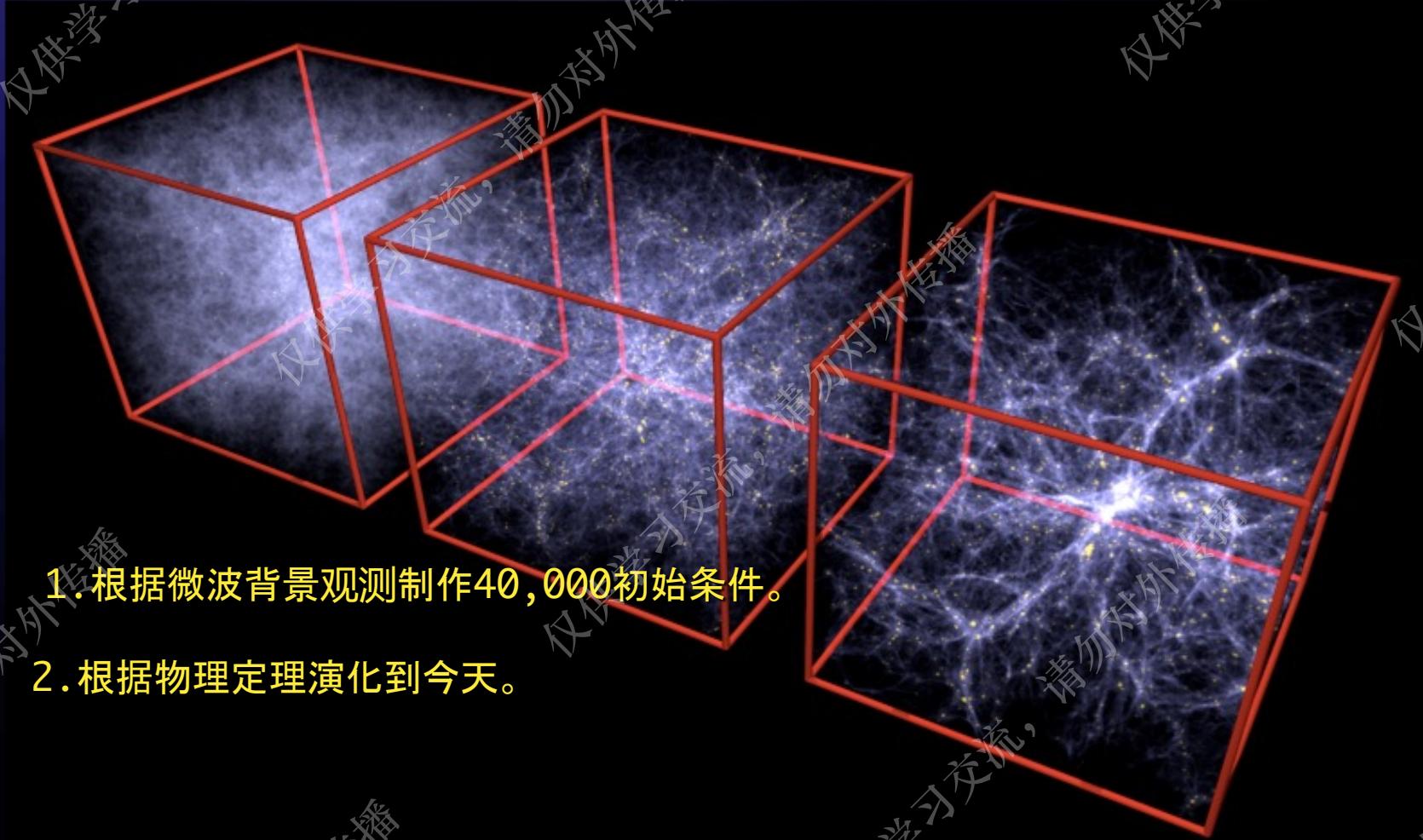
### 能量方程

$$\frac{du}{dt} = -\frac{P}{\rho} \nabla \cdot \mathbf{v} - \frac{\Lambda(u, \rho)}{\rho}$$

宇宙原初条件可以从宇宙微波背景辐射获得



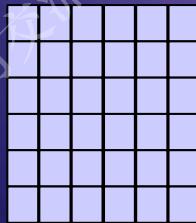
# 用计算机重演宇宙演化



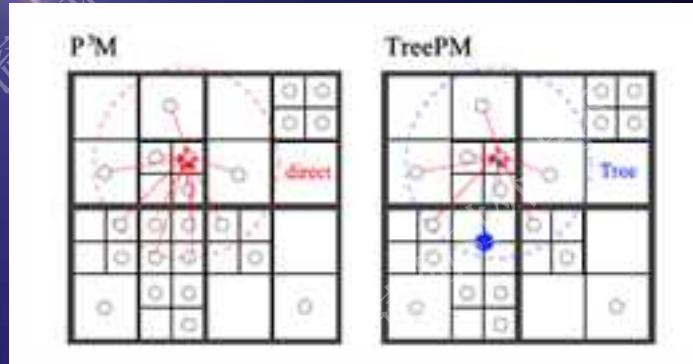
1. 根据微波背景观测制作40,000初始条件。
2. 根据物理定理演化到今天。

# 引力计算

- Particle Mesh (PM)
- Particle-Particle PM (P3M)
- Tree ; Tree-PM (Gadget)
- FMM (线性可扩展性)

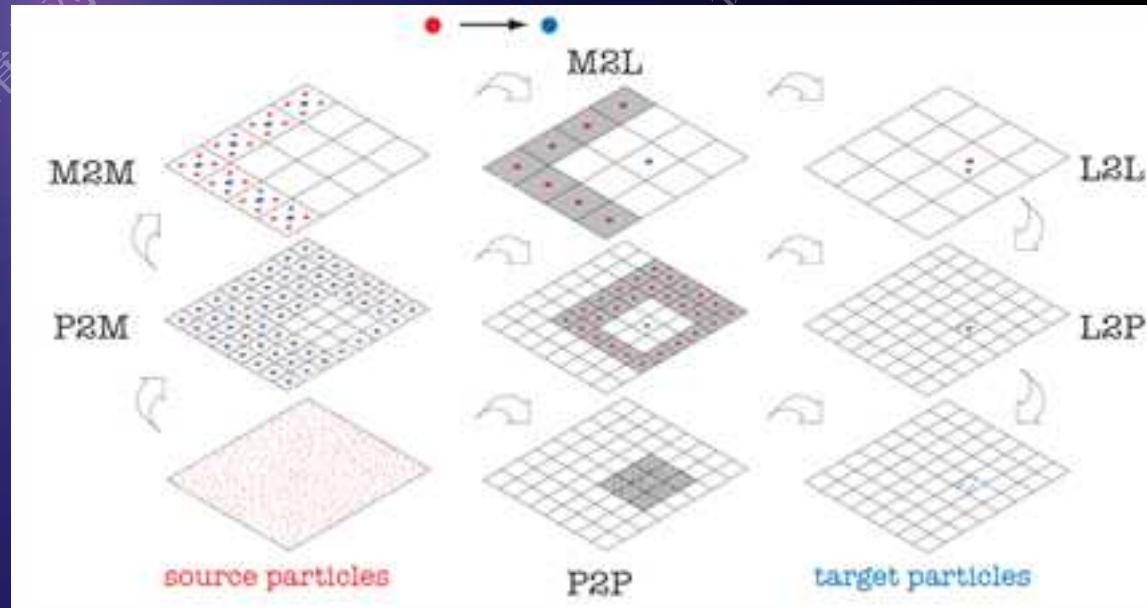
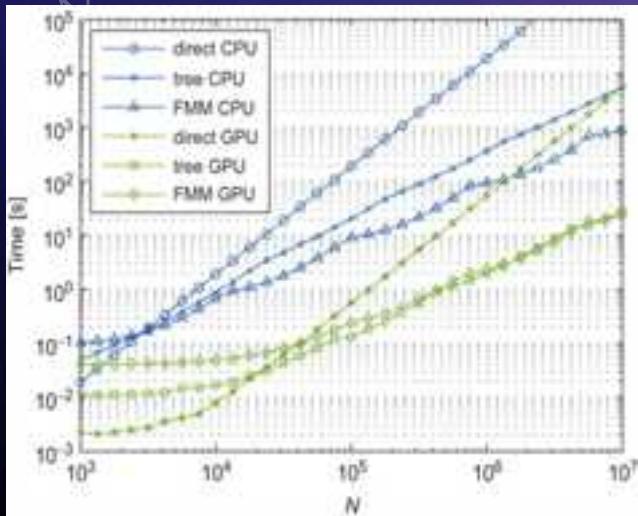


泊松方程在傅里叶空间存在格林函数  
可以利用快速傅里叶变换



# Photons

- 当  $N > 10^{10}$ , FMM 效率相对流行的 Tree-PM 高

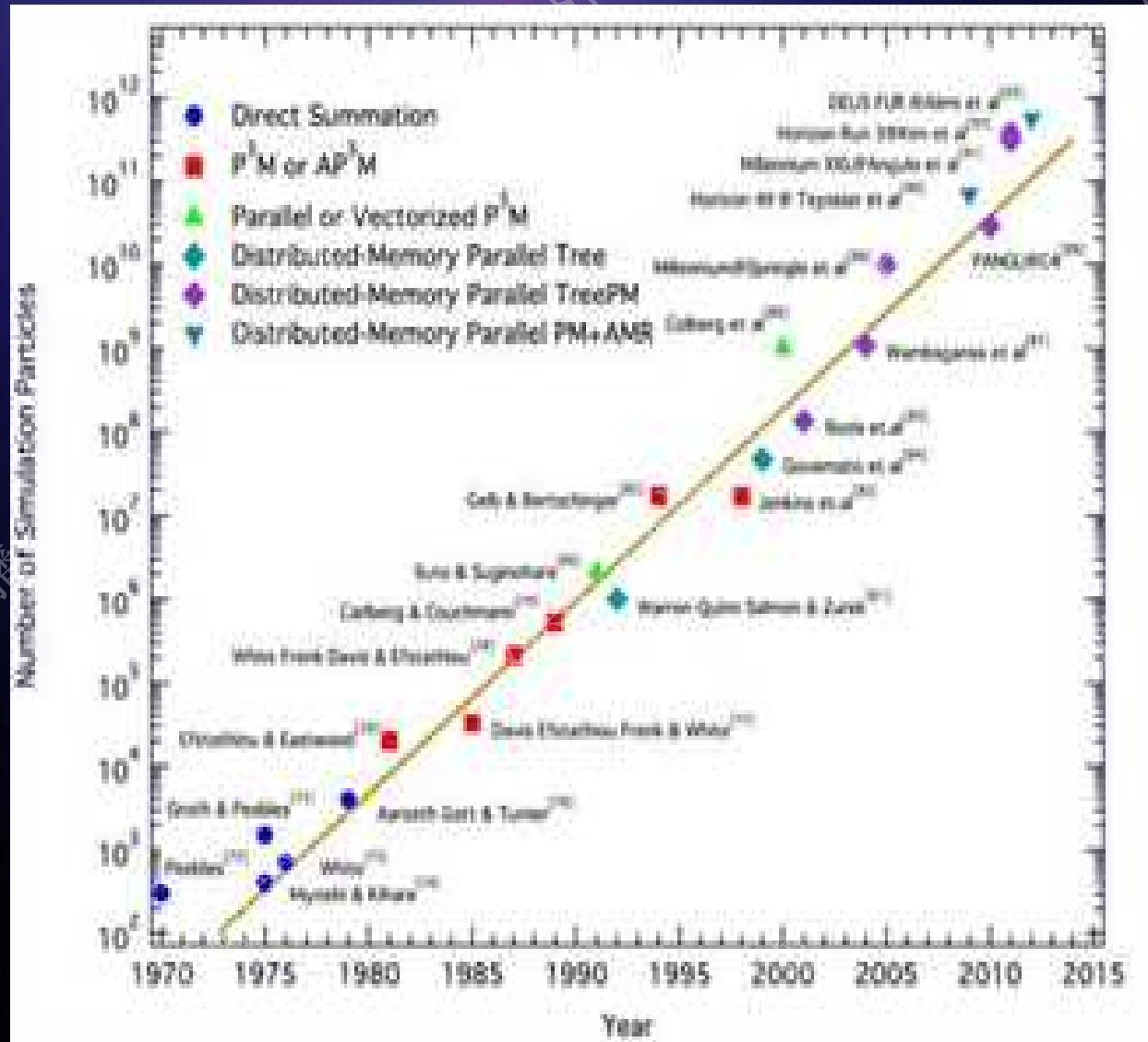


Yokota & Barba 2011

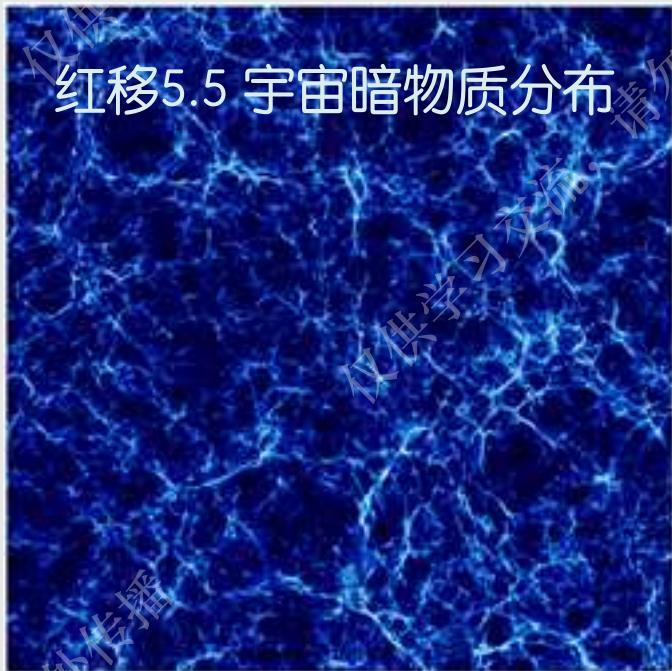
FMM 是  $O(N)$  的算法。在超大规模的情况，可扩展性更好

- 采取 FMM+PM 混合求解泊松方程
- 90% 算力在加速卡

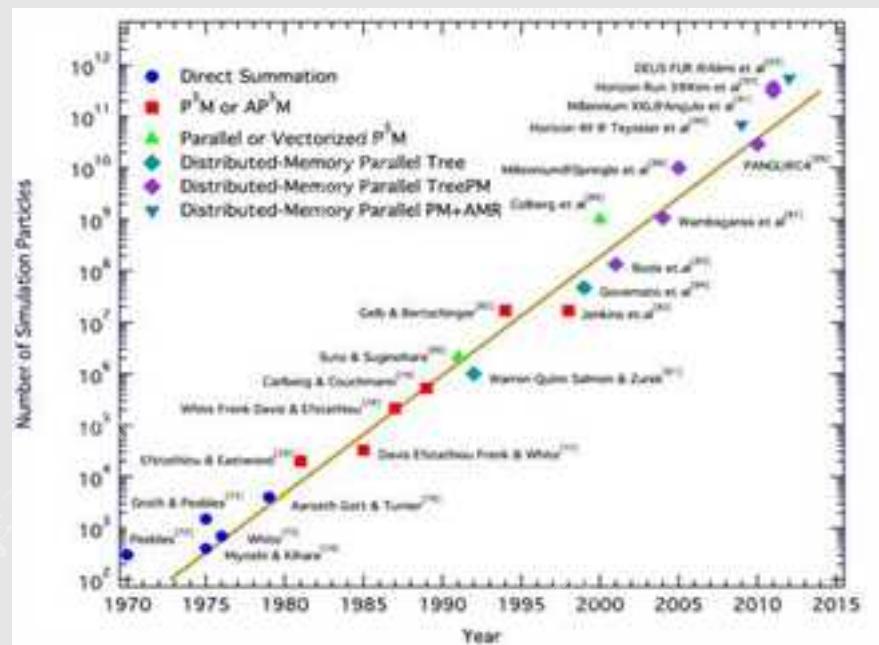
王乔, 高亮等, 2019



# 世界最大规模、最高精度宇宙再电离期宇宙学模拟—群青



粒子最大规模 vs 年代 Ultramarine

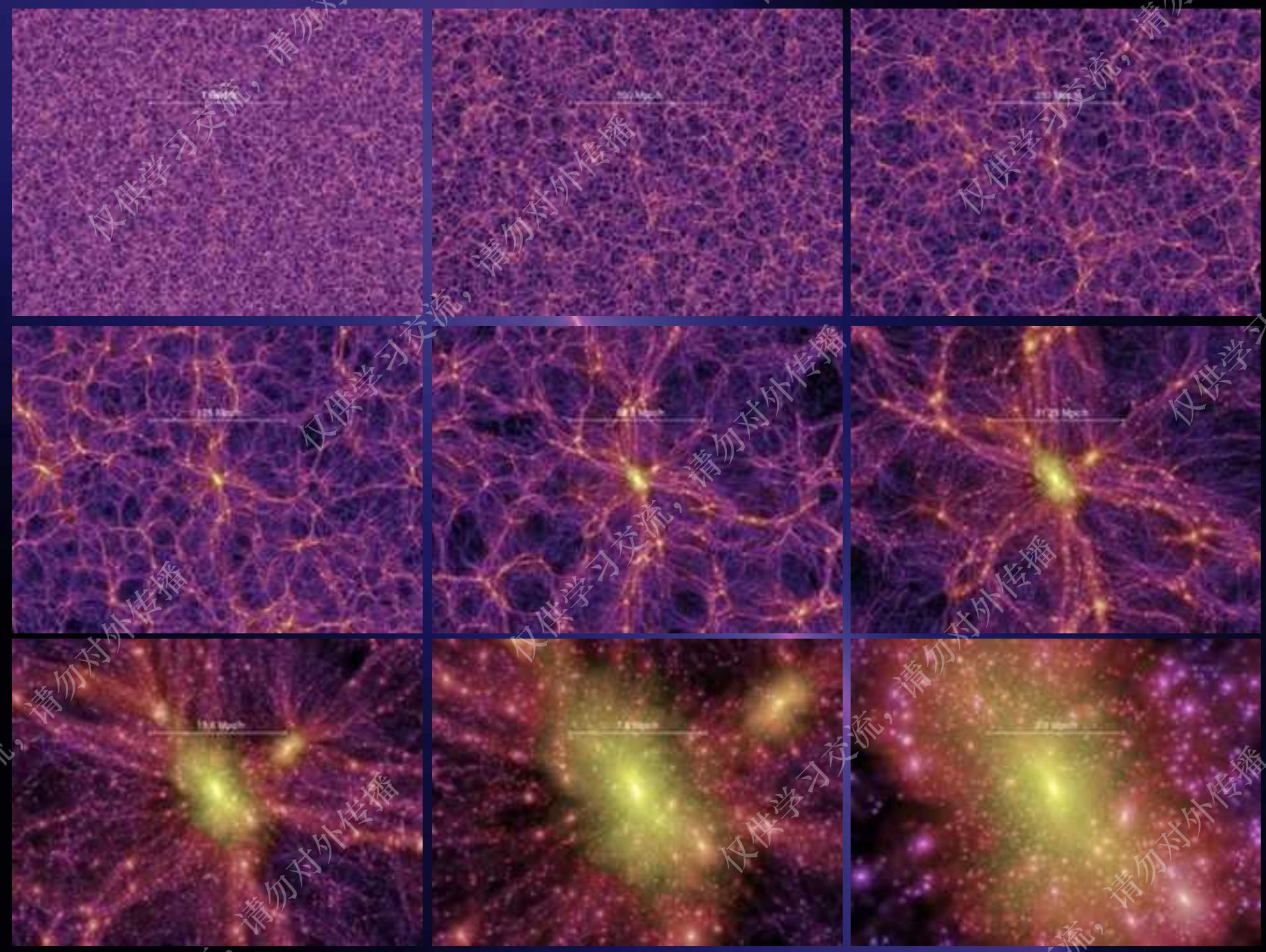


A cluster of three yellow star emojis, with two visible in the foreground and one partially visible behind them.

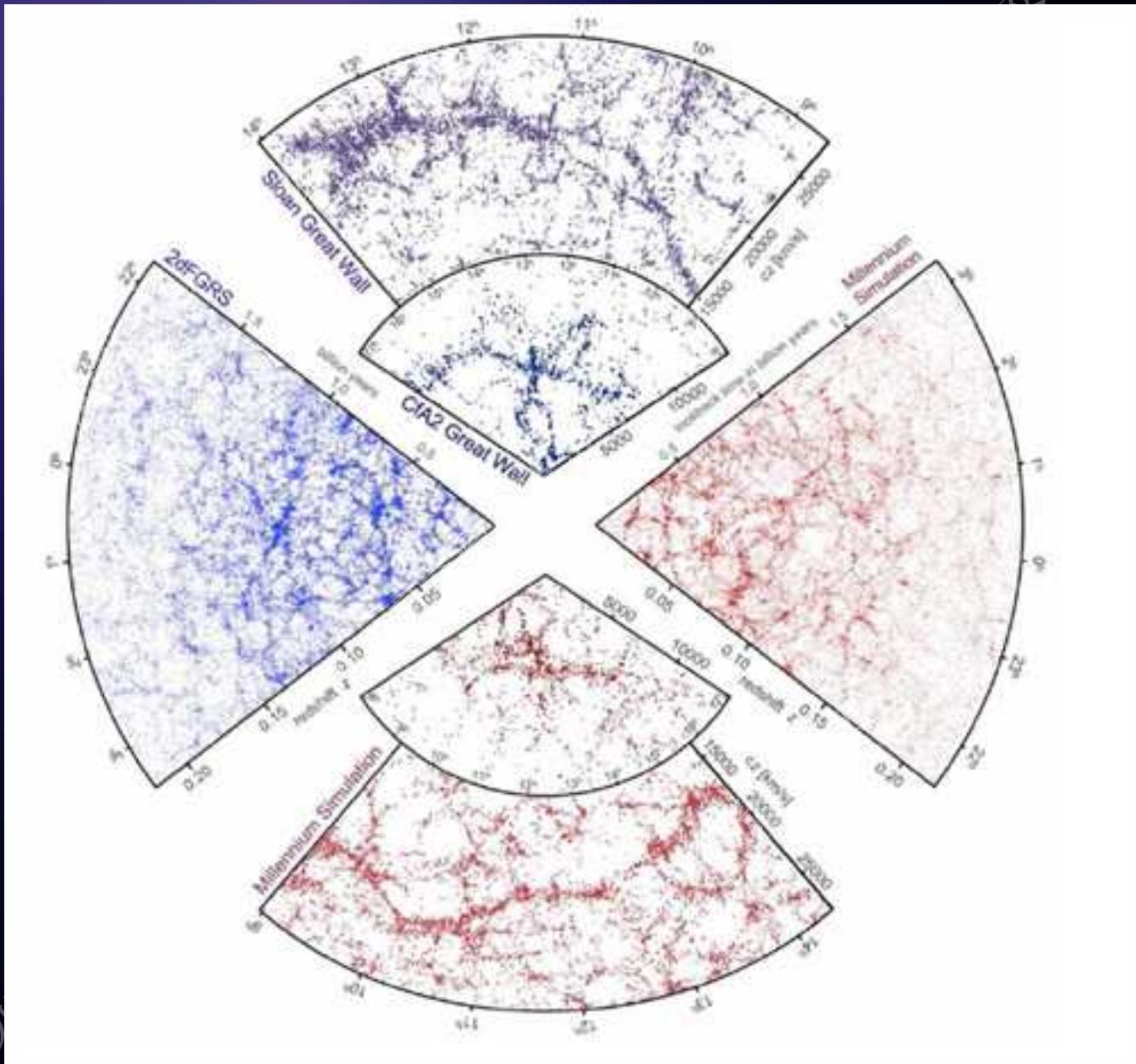
群青

- 粒子数2万亿，质量分辨率500万太阳质量
  - 2倍世界纪录模拟预计本月完成





# 虚拟 v.s. 观测



# 暗物质晕

A rich galaxy cluster  
halo Gao et al. 2012



$10^{12}$  太阳质量

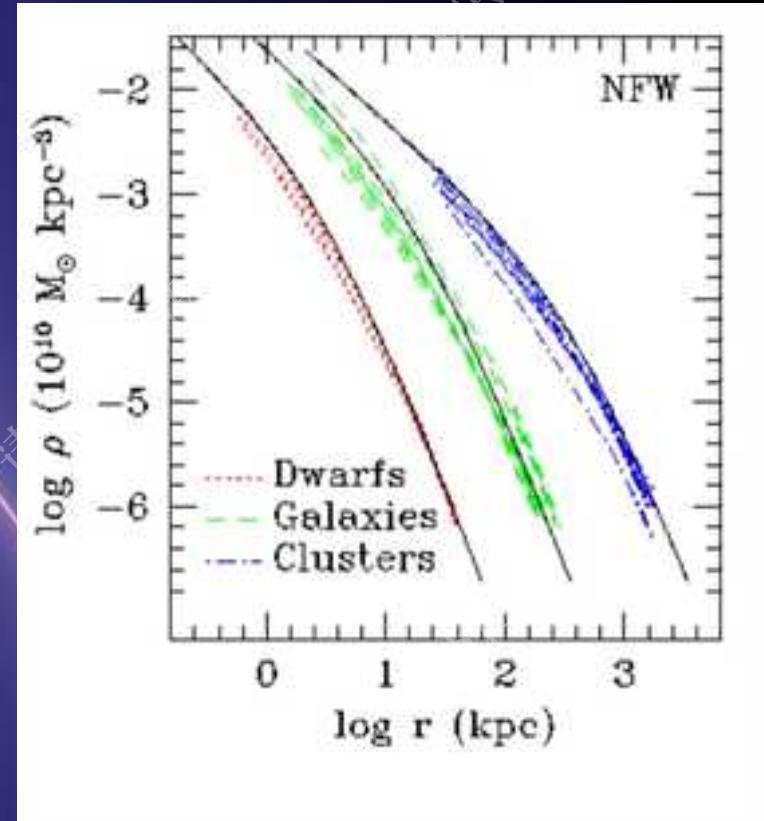
$10^{15}$  太阳质量

# Density profiles of dark matter halos

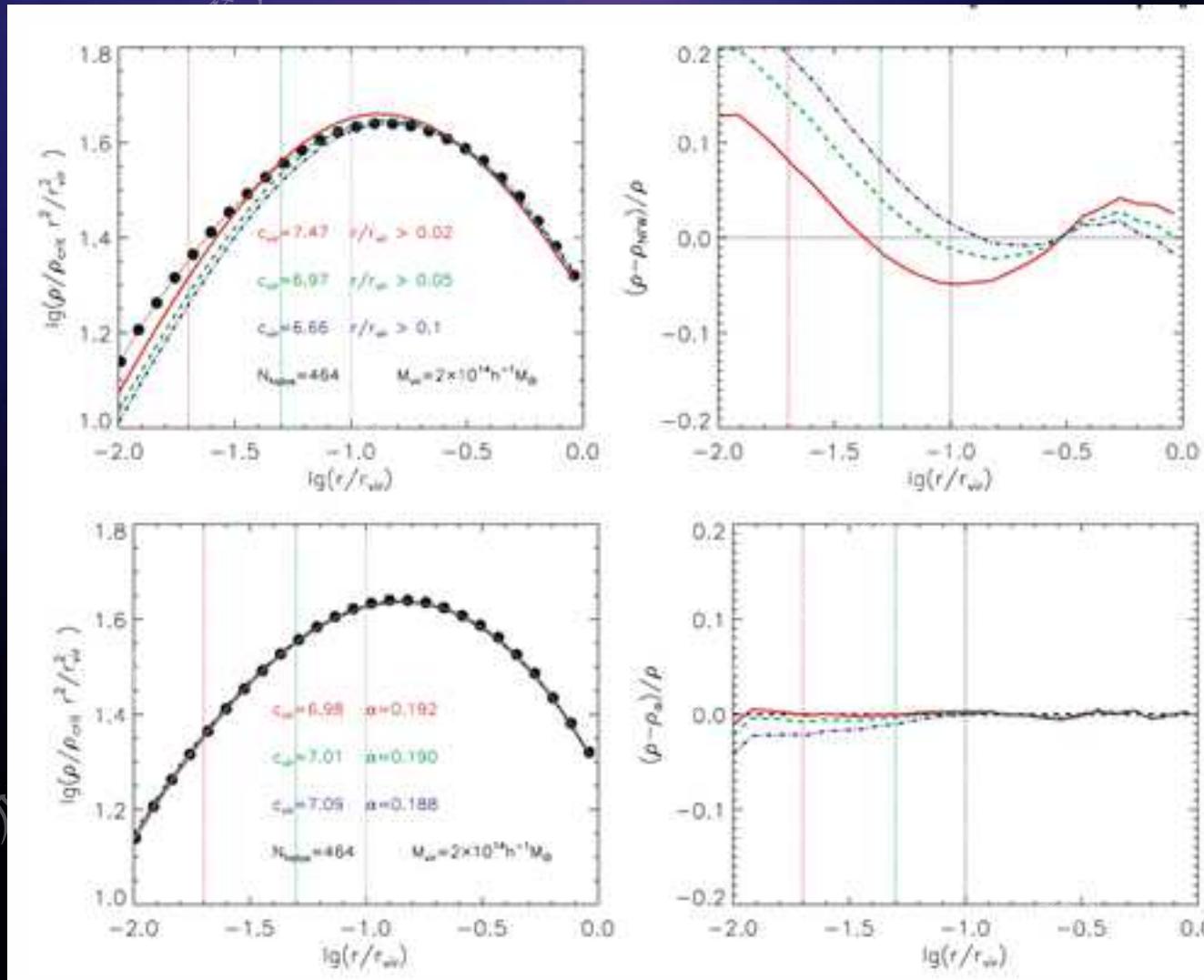
$$\rho(r)/\langle \rho \rangle \approx \delta r_s / r(I + r/r_s)^2$$

Navarro et al. 1996

Resolution ~ 10000 Particles per halo



# NFW v.s Einasto



Gao et al. 2008

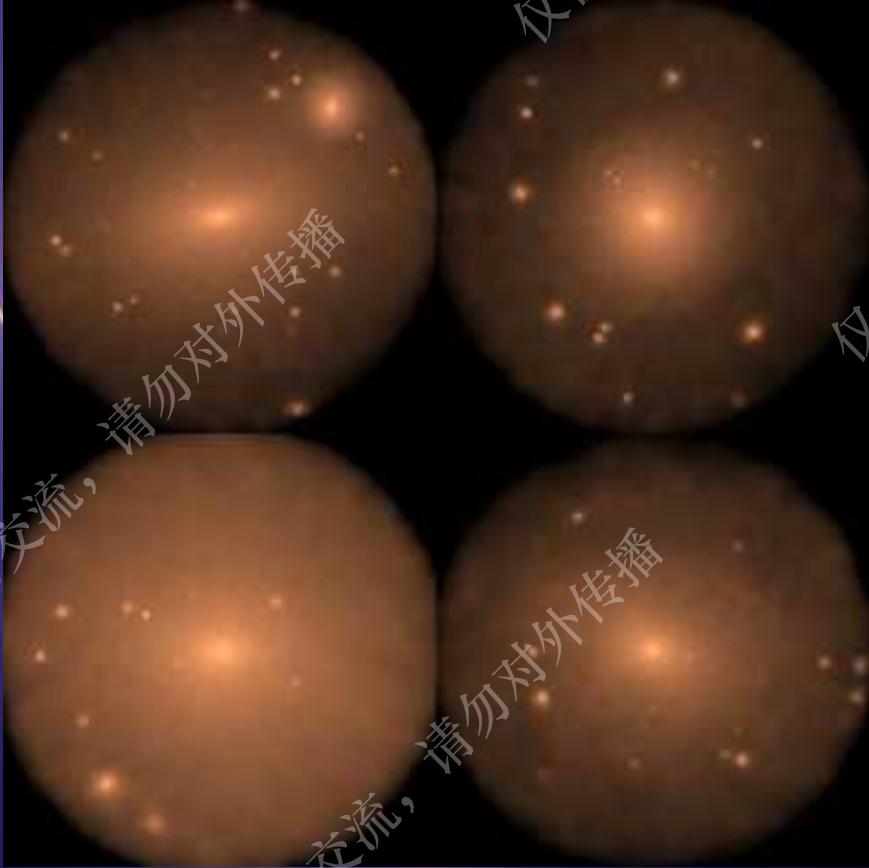
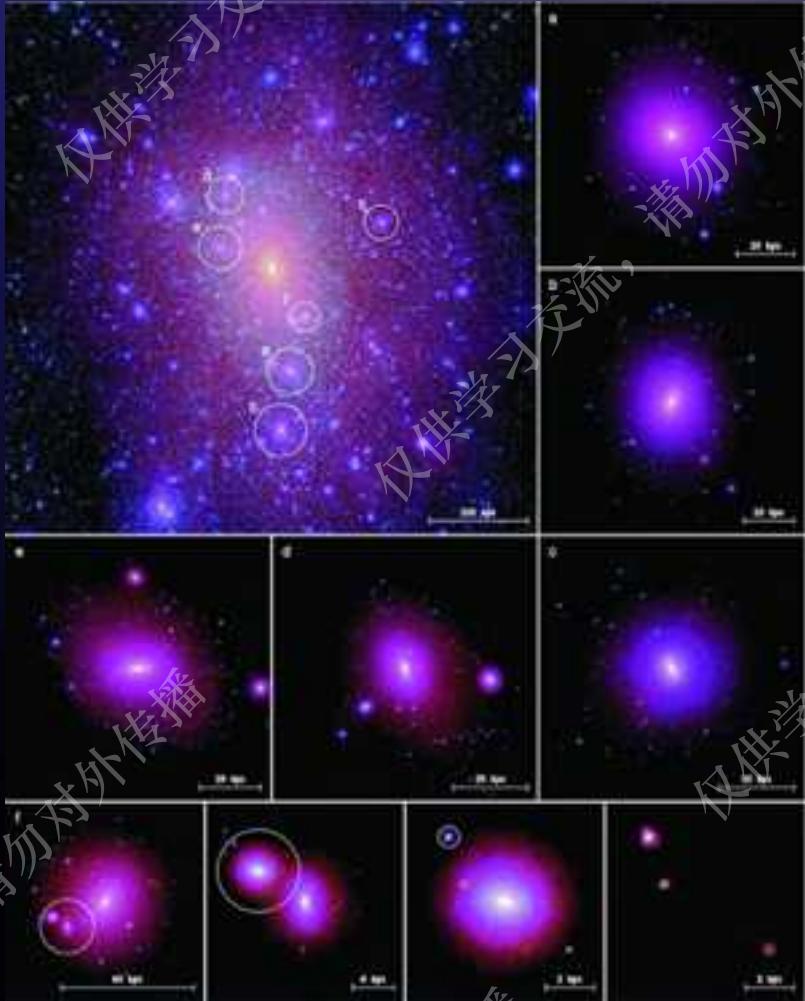
1e5 particles  
per halo

Einasto fits:

$$\ln\left(\frac{\rho}{\rho_{-2}}\right) = -\left(\frac{2}{\alpha}\right) \left[\left(\frac{r}{r_{-2}}\right)^a - 1\right]$$

a is a free para

Navarro et al. 2003



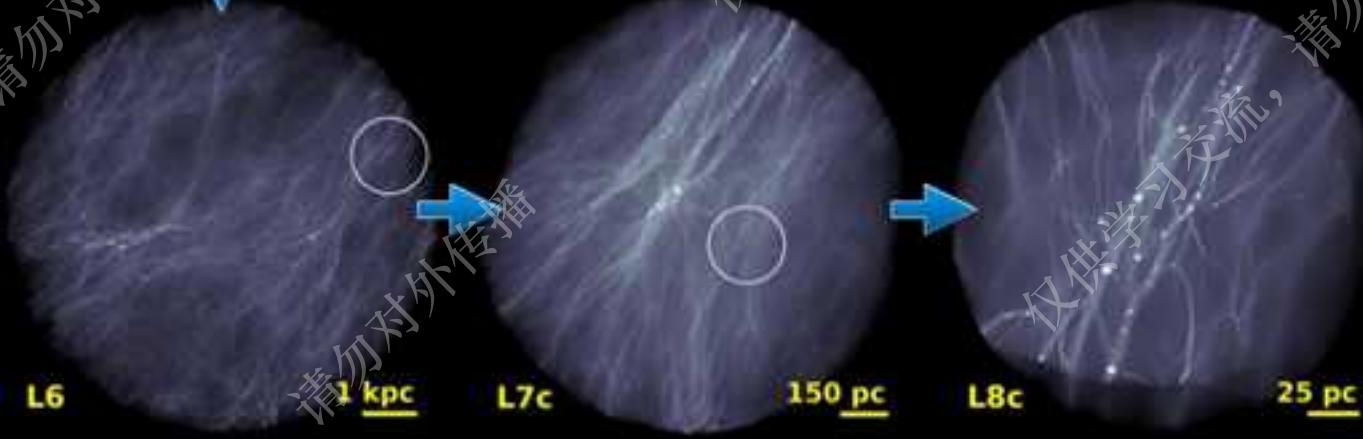
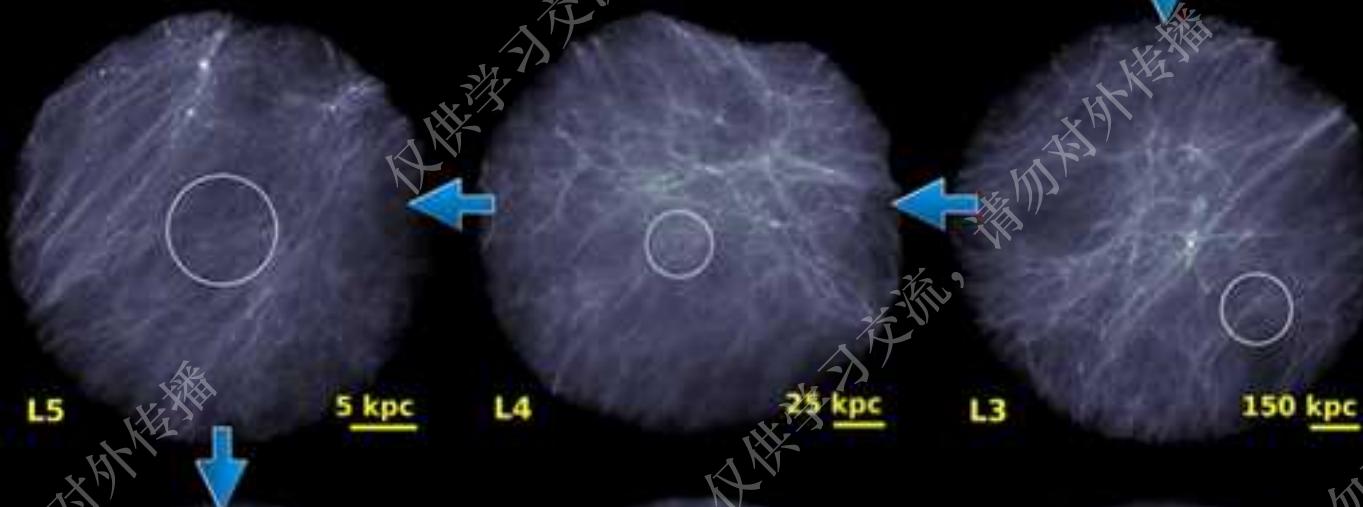
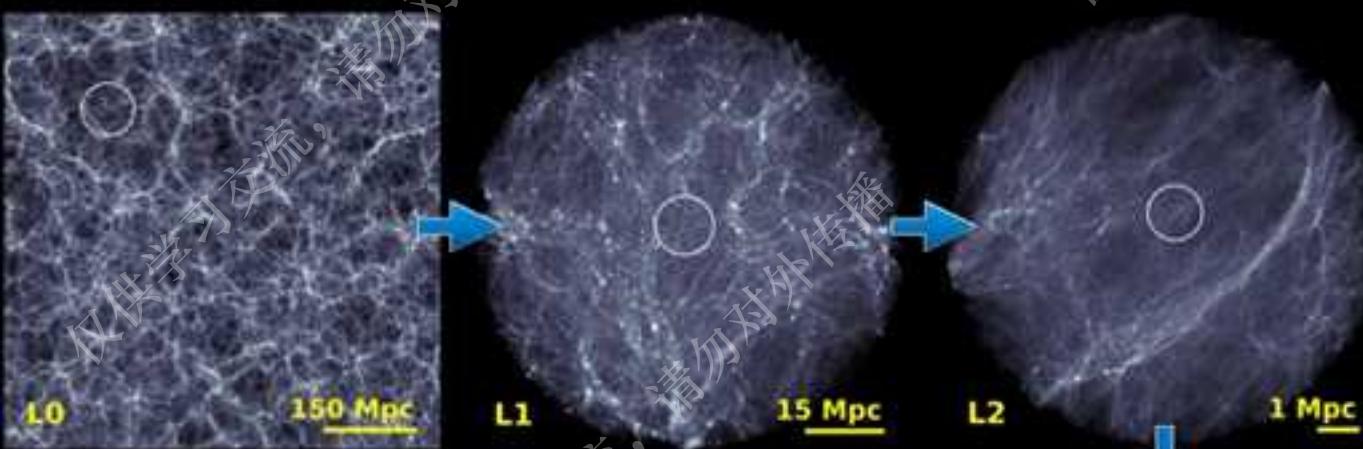
子结构又含子-子结构。...

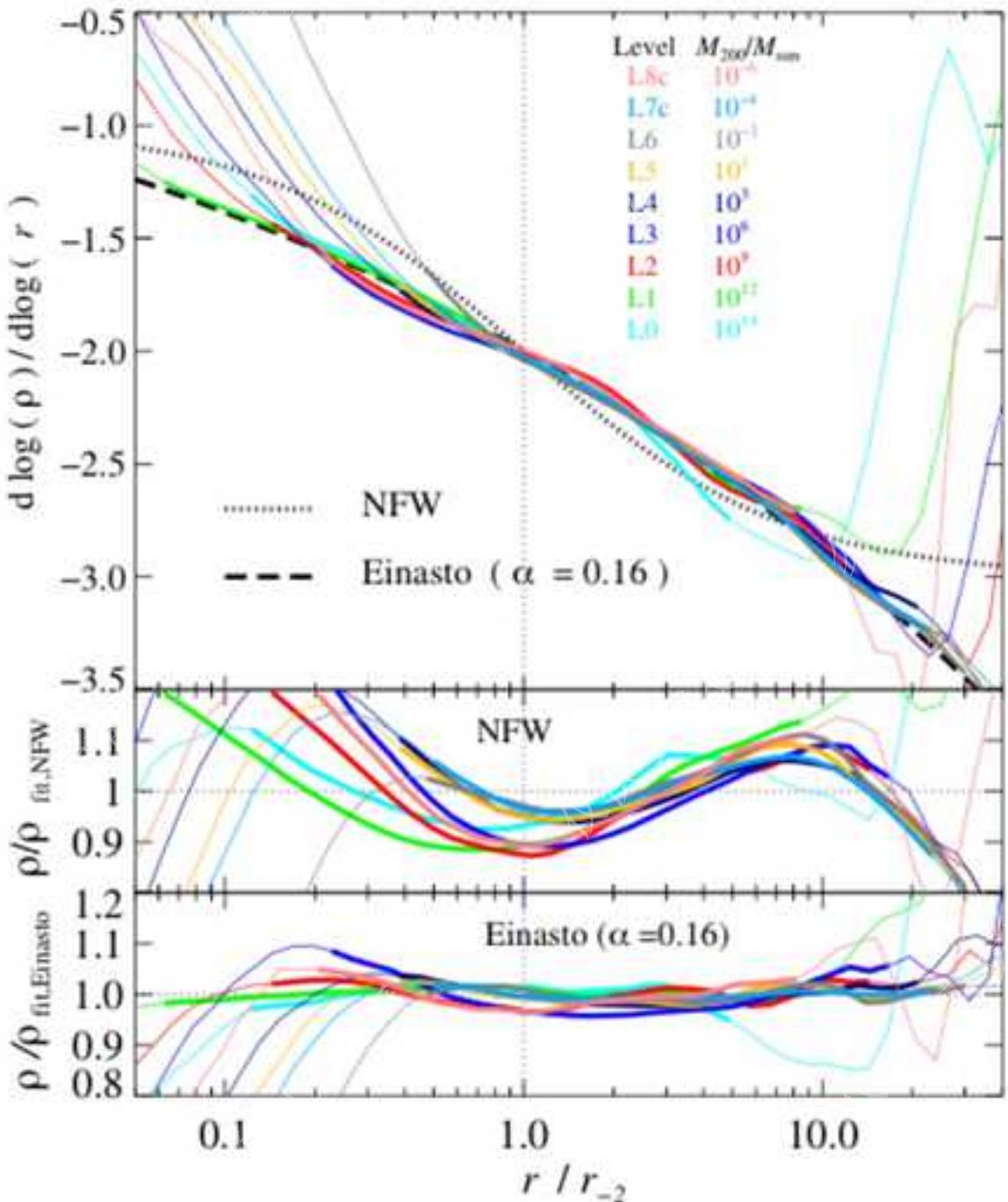
通过八级

Zoom-in

模拟

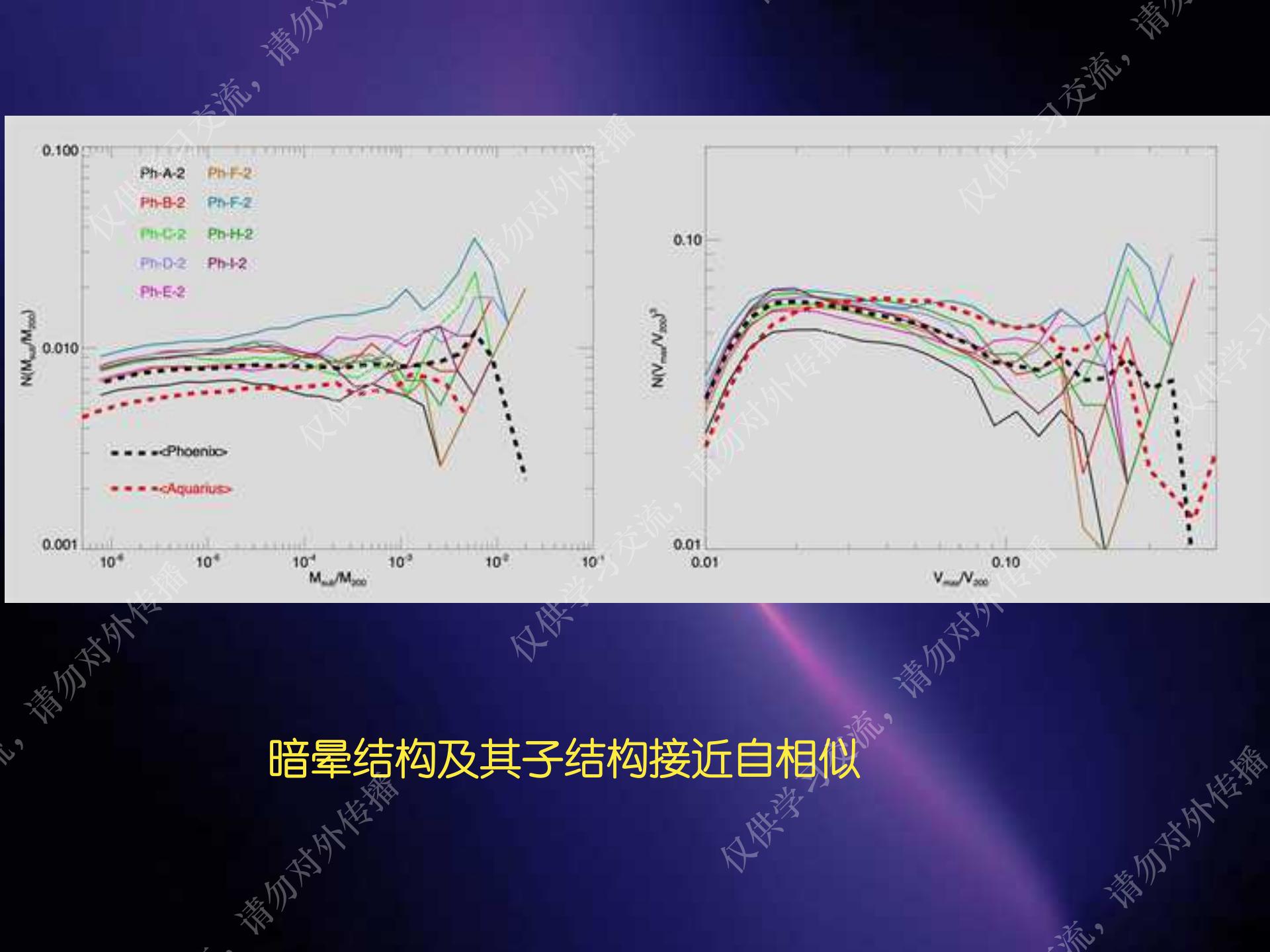
解析出所有  
质量的暗晕的结  
构



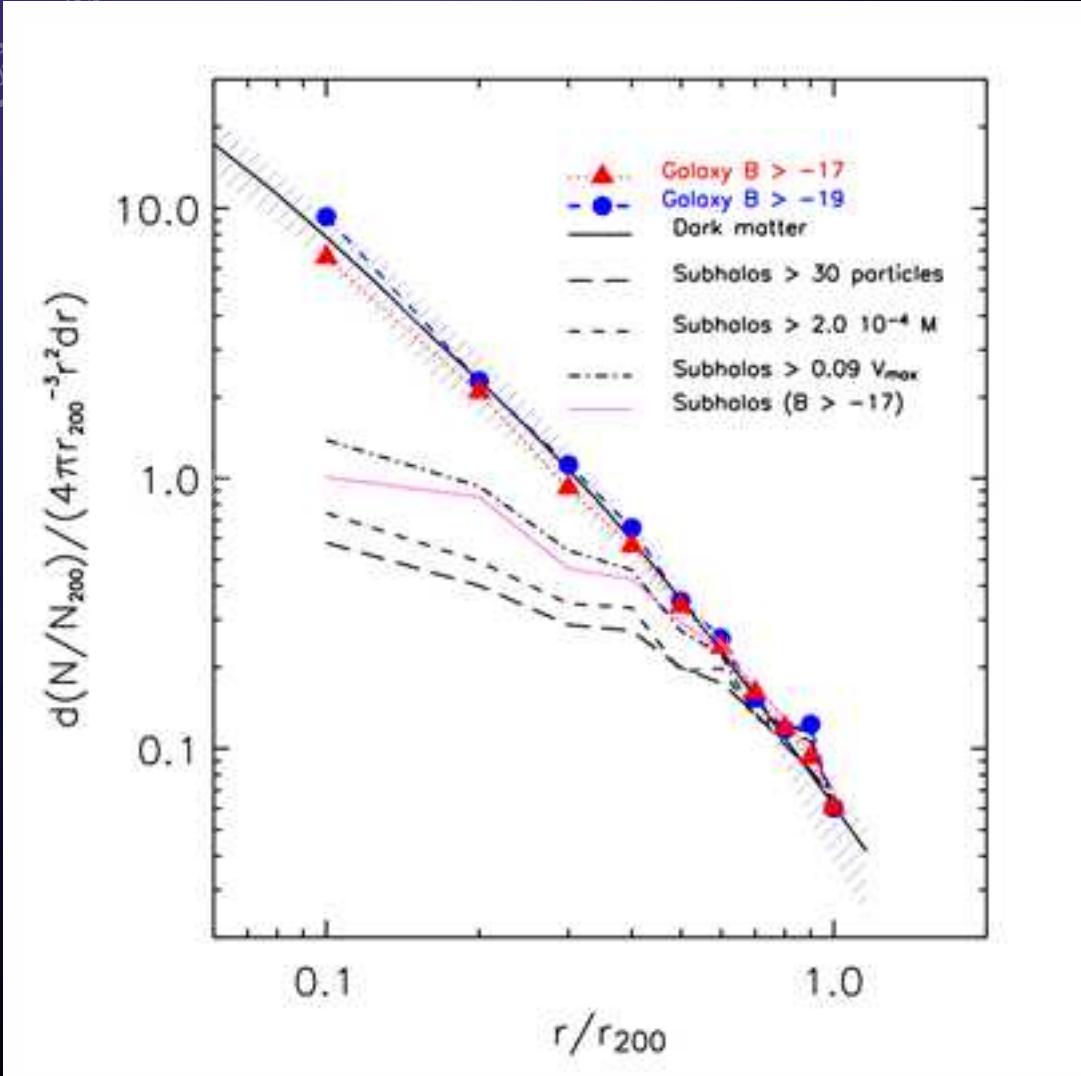


结宇宙所有质量  
暗晕构高度相似

Wang et al.  
2020, Nature



暗晕结构及其子结构接近自相似

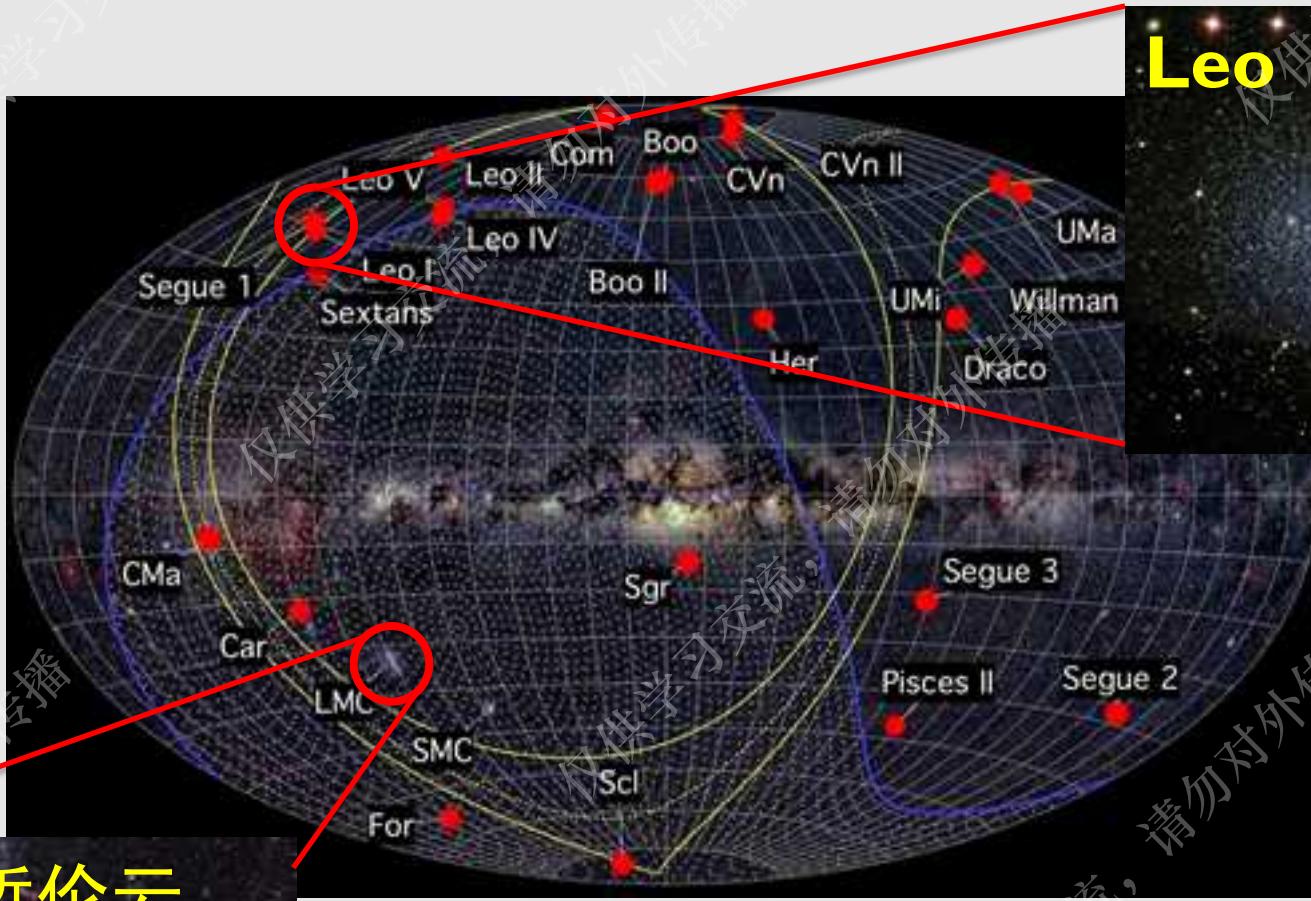


暗晕子结构分布比暗物质 less concentrated

# 宇宙结构最基本单元—暗物质晕 冷暗物质预言

- 暗晕密度轮廓是尖的
- 暗物质晕包含相当多的子结构
- 暗晕可近似为自相似 scale free

## Leo I 矮椭星系

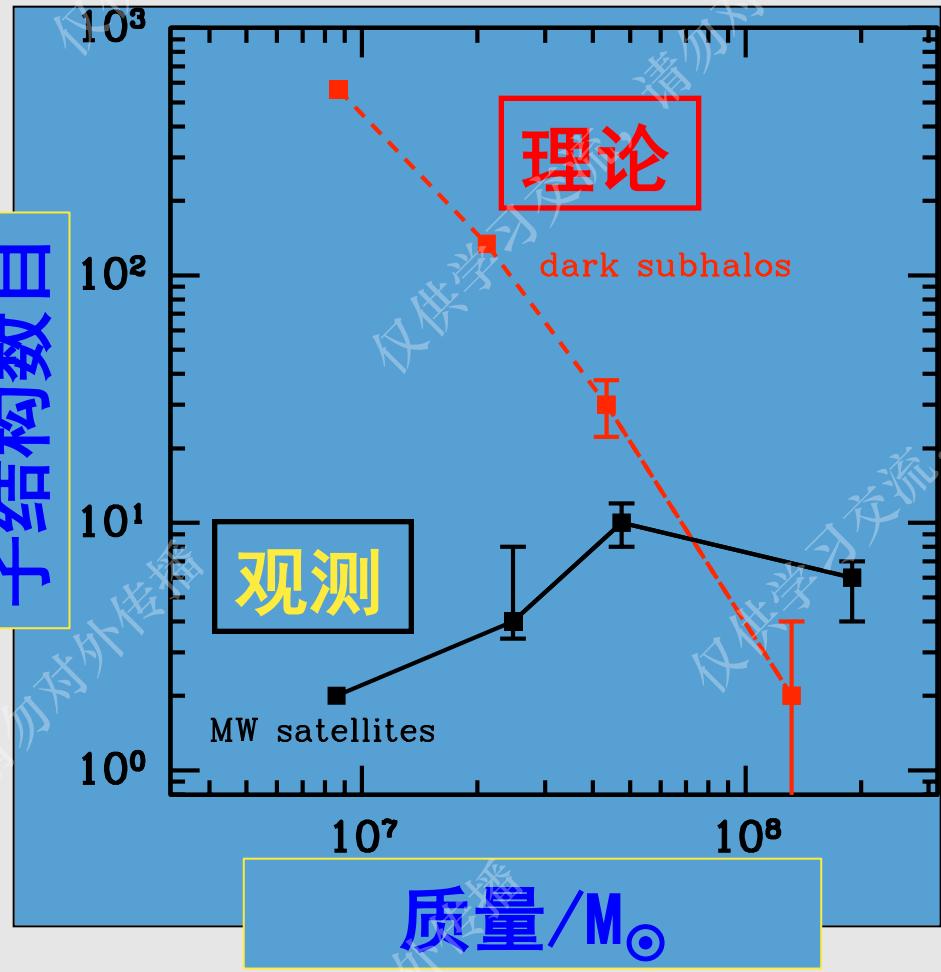


大麦哲伦云

# The missing satellite problem

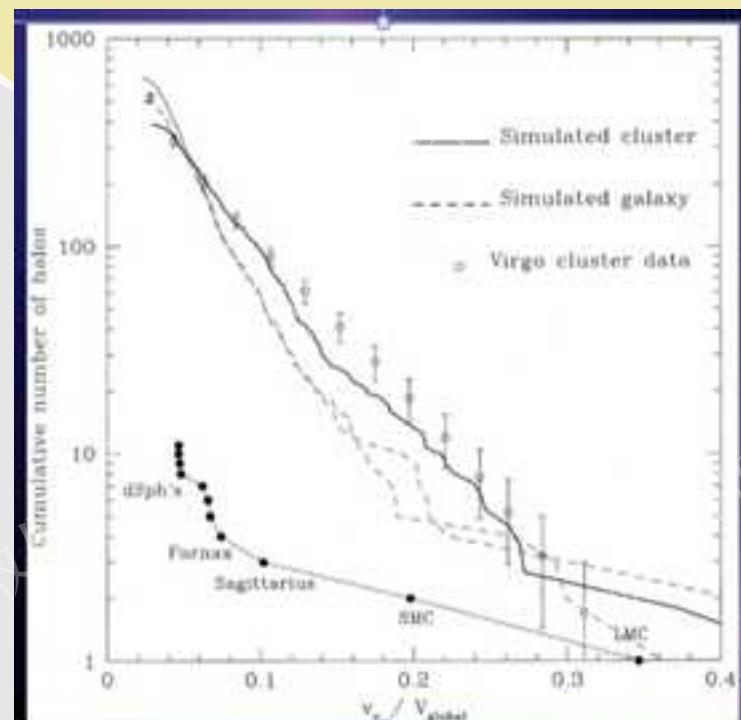
可见子结构数目

子结构数目

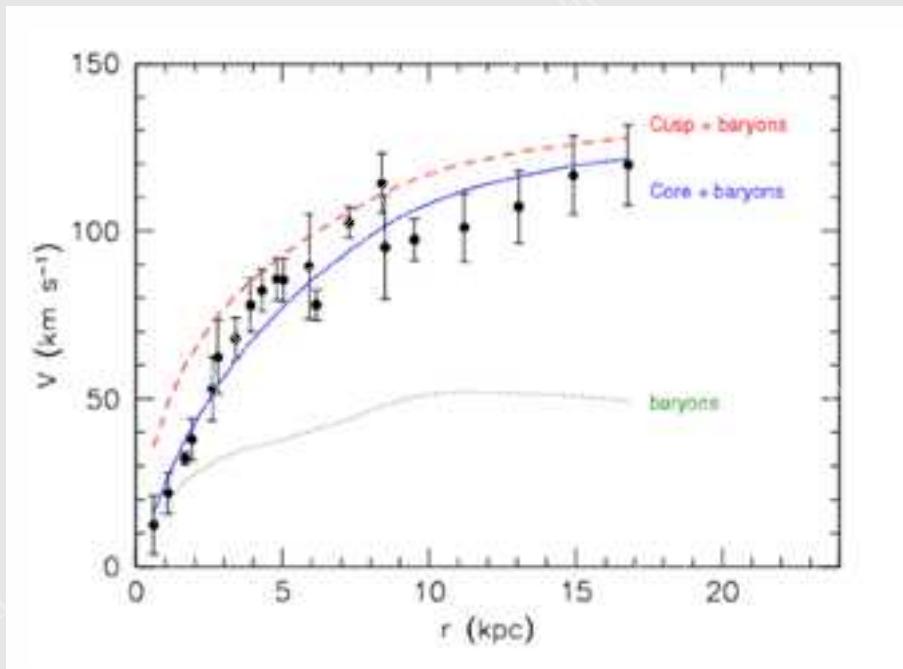


Moore et al. 1996

冷暗物质模型预言的子结构数目



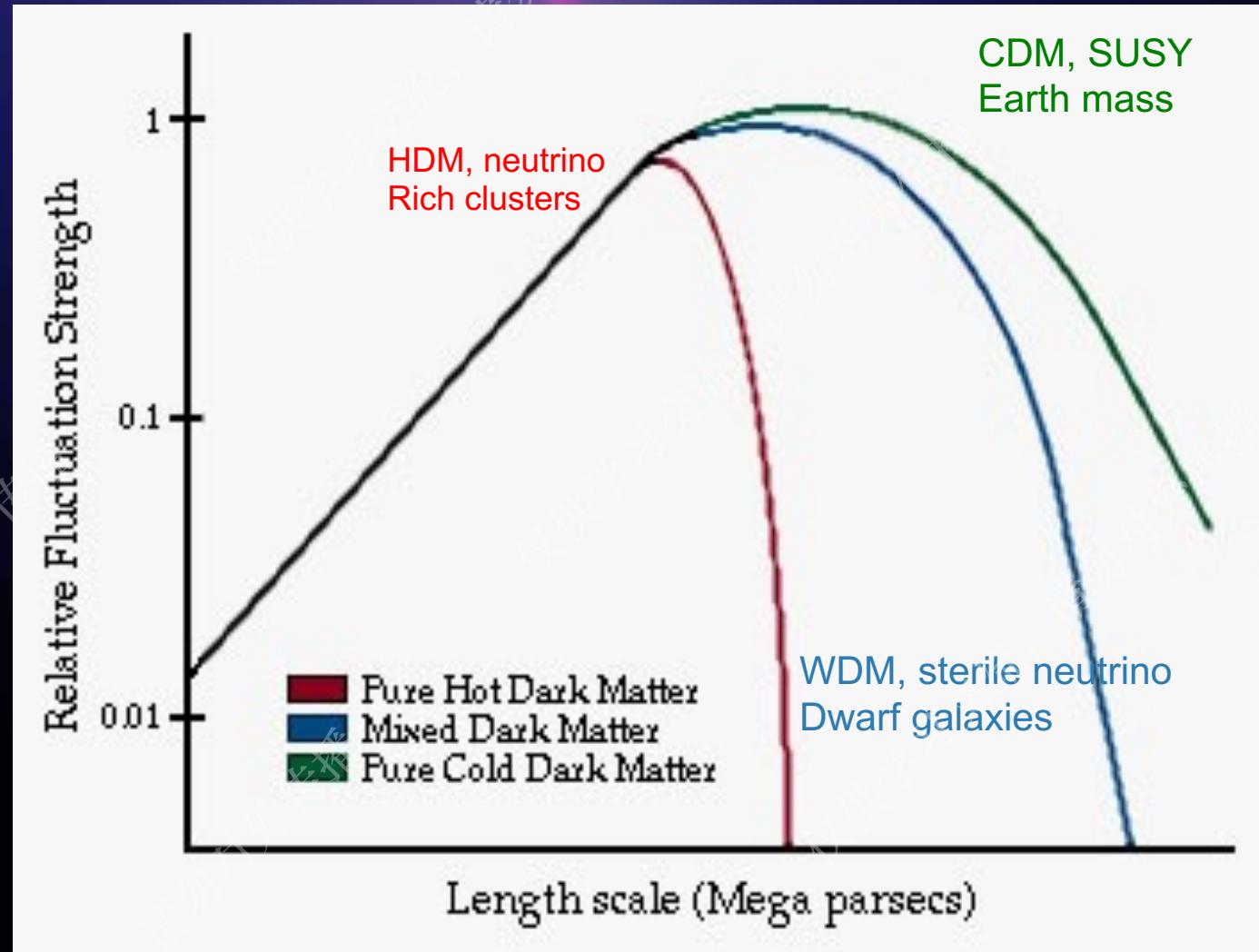
# 矮星系内部密度结构问题



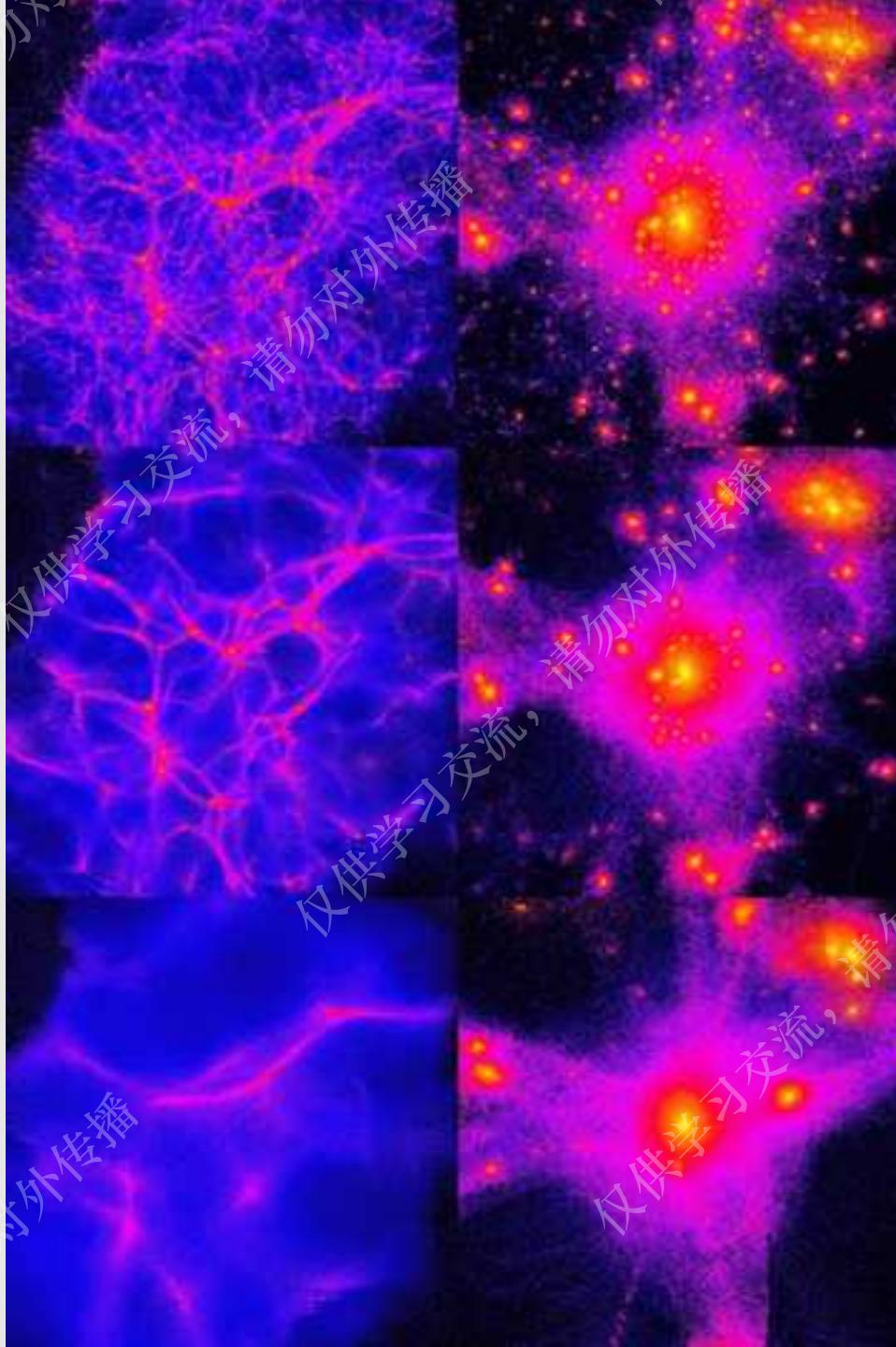
Kuzio de Naray et al 08

Core/Cusp problem

# 宇宙结构形成历史由宇宙原初功率，而 原初功率谱由暗物质属性决定



## 不同暗物质 模型对宇宙 结构形成的影响



冷暗物质

温暗物质

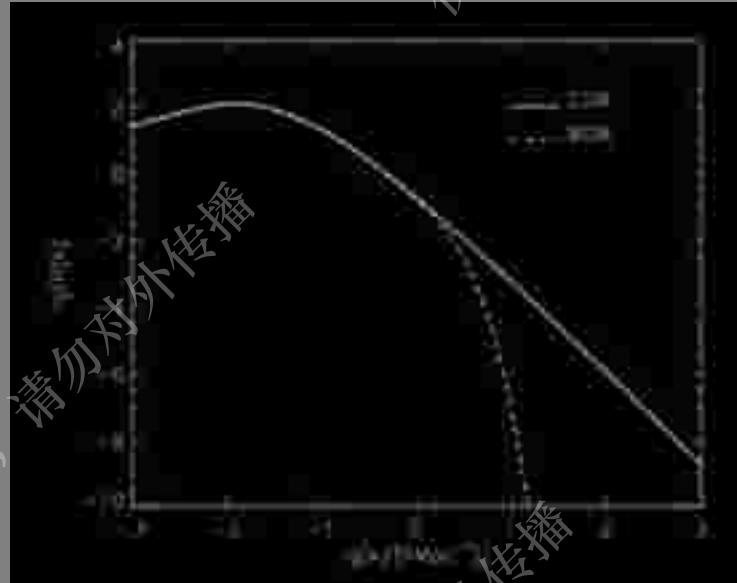
热暗物质

# 温暗物质模型独有特征

Particles free streaming



Power spectrum



Momentum distribution  $f_n$

$$f_{FD}(p) = \frac{g}{(2\pi\hbar)^3} \frac{1}{e^{(E-\mu)/T_D} + 1}$$



Cored (PSD constrain)



To solve cusp/core problem

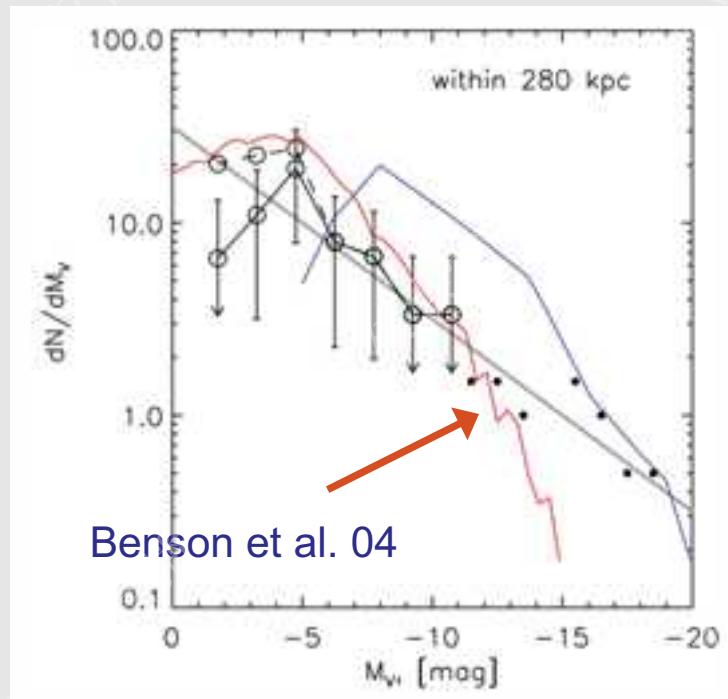


Reduces low mass halo abundance



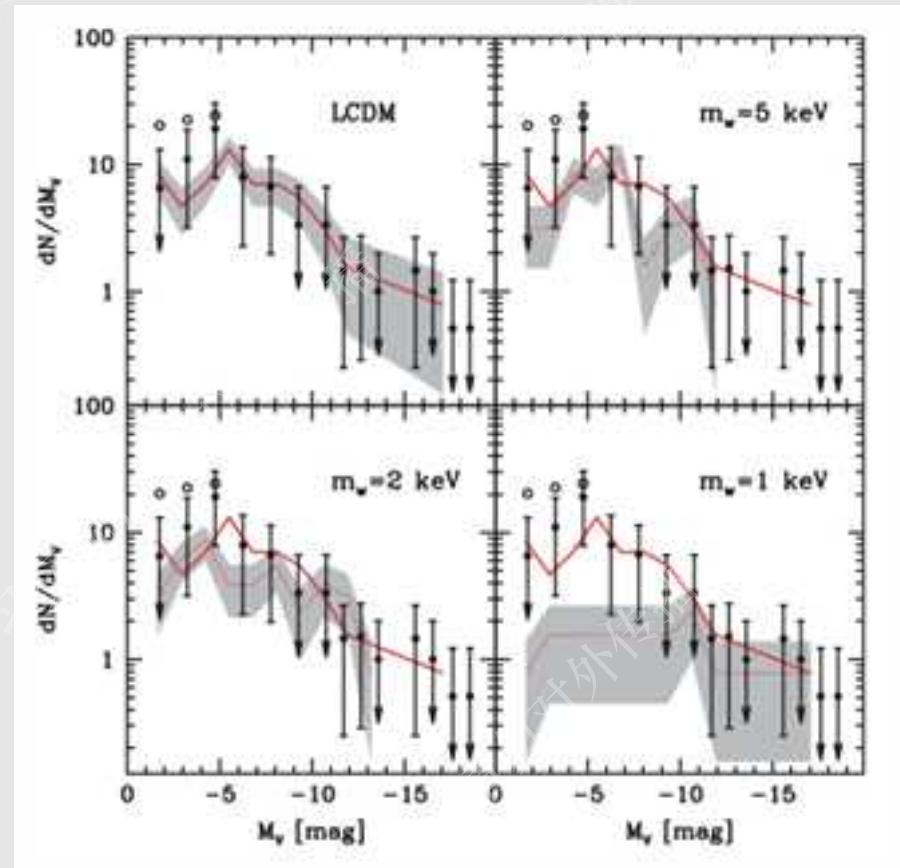
To solve MW satellites problem

# 银河系卫星星系缺失问题



Koposov et al. 2008

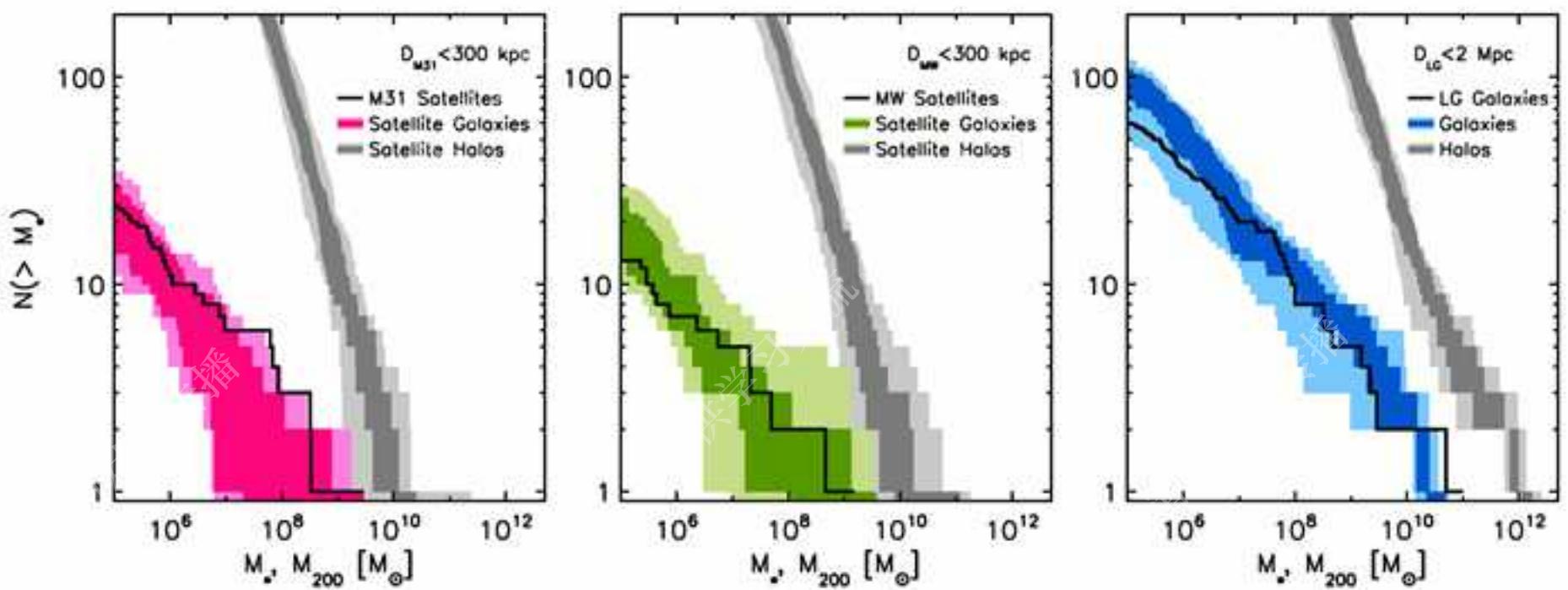
宇宙再电离，加热气体，  
小暗晕无法发生星系形成



Maccio et al. 2010, SAM

WDM does a good job too!

# Missing satellites problem



Apostle simulation

Sawala et al. 2016

# Fine-grained Phase Space Density Bound

- Warm dark matter candidates, e.g. thermal relics, their moment distribution follow a Fermi-Dirac distribution (Tremaine & Gunn 1979).

$$f_{FD}(p) \approx \frac{g}{(2\pi\hbar)^3} \frac{1}{e^{pc/T_D} + 1}$$

- We can derive the density

$$\rho = m_x \times n = \frac{gm_x}{(2\pi\hbar)^3} \int \frac{d^3 p}{e^{pc/T_D} + 1} \leq \frac{gm_x^4}{2(2\pi\hbar^3)} \int d^3 v$$

- upper bound of fine-grained phase space density is

$$f_{FD} = \frac{gm_x^4}{2(2\pi\hbar)^3}.$$

# Maximal coarse-grained Phase space density estimation

- Assume central region of warm dark matter halos have a pseudo-isothermal profile

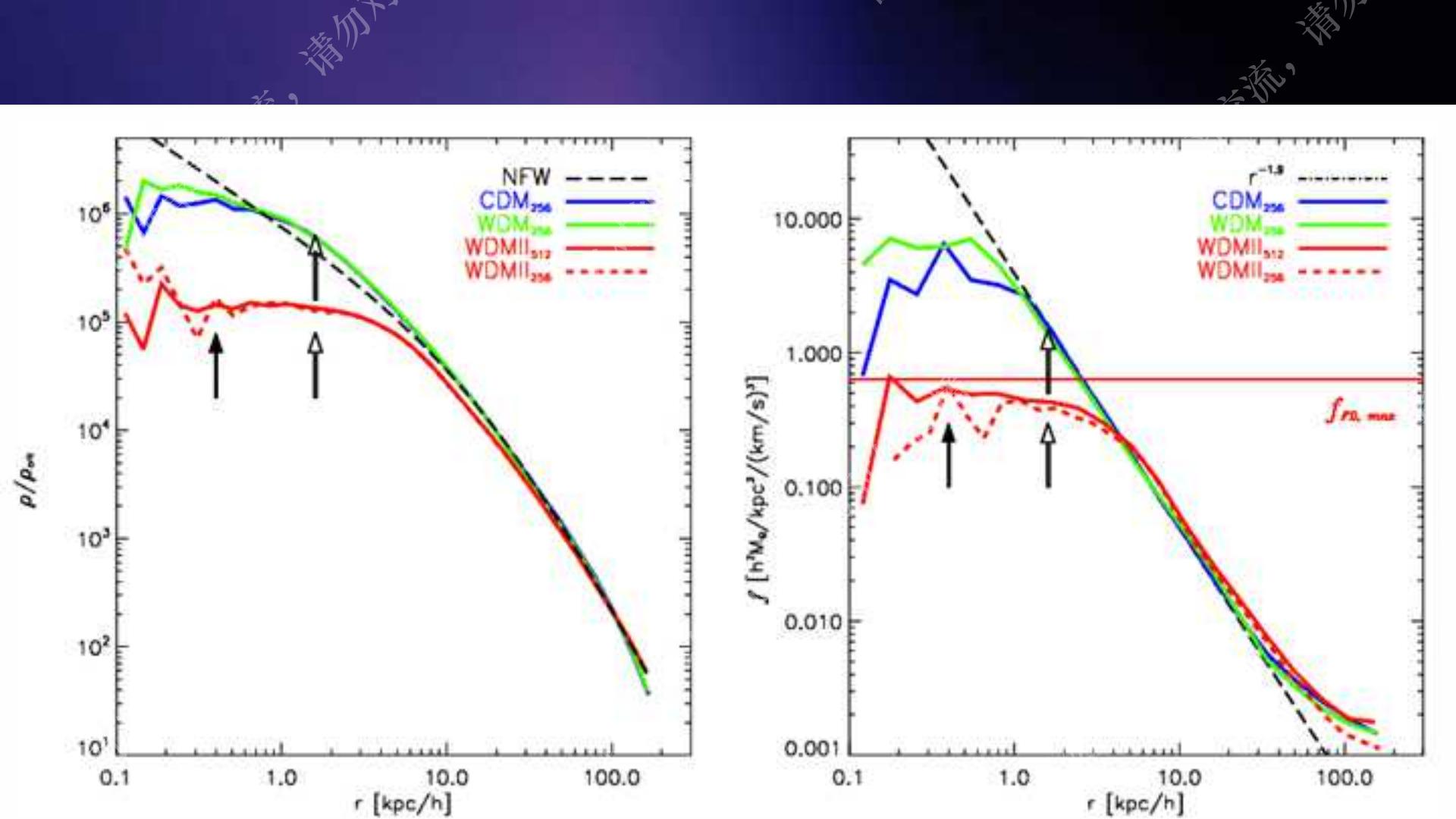
$$\rho(r) = \frac{\rho_0}{[1 + (\frac{r}{r_c})^2]}.$$

- Central density, core radius and velocity dispersion is related as

$$\rho_0 = \frac{1}{2\pi G} \frac{\sigma^2}{r_c^2}.$$

- Coarse-grained maximum phase space density as

$$f_0 = \frac{\rho_0}{(2\pi\sigma^2)^{3/2}} = \frac{1}{(2\pi)^{5/2}G} \frac{1}{\sigma r_c^2}.$$



## Real and phase space density profiles

# Application to real data

- Assume the stellar and dark matter component has the same core radius in dSphs

$$f_0 = \frac{\rho_0}{(2\pi\sigma^2)^{3/2}} = 7.05 \frac{M_\odot/\text{pc}^3}{(\text{km s}^{-1})^3} \left( \frac{\text{km/sec}}{\sigma} \right) \left( \frac{1\text{pc}}{r_h} \right)^2. \quad (11)$$

For the pseudo-isothermal profile, its projected surface density  $S(R)$  can be written as:

$$S(R) = \int_{-\infty}^{\infty} \rho[(R^2 + z^2)^{1/2}] dz = \frac{S_0 r_c}{\sqrt{r_c^2 + R^2}} \quad (12)$$

where  $R$  is projected radius,  $S_0 = \pi\rho_0 r_c$  is central surface density. Hence the half projected surface density  $r_h$  can be related to half density  $r_c$  as,  $r_h = \sqrt{3}r_c$ .

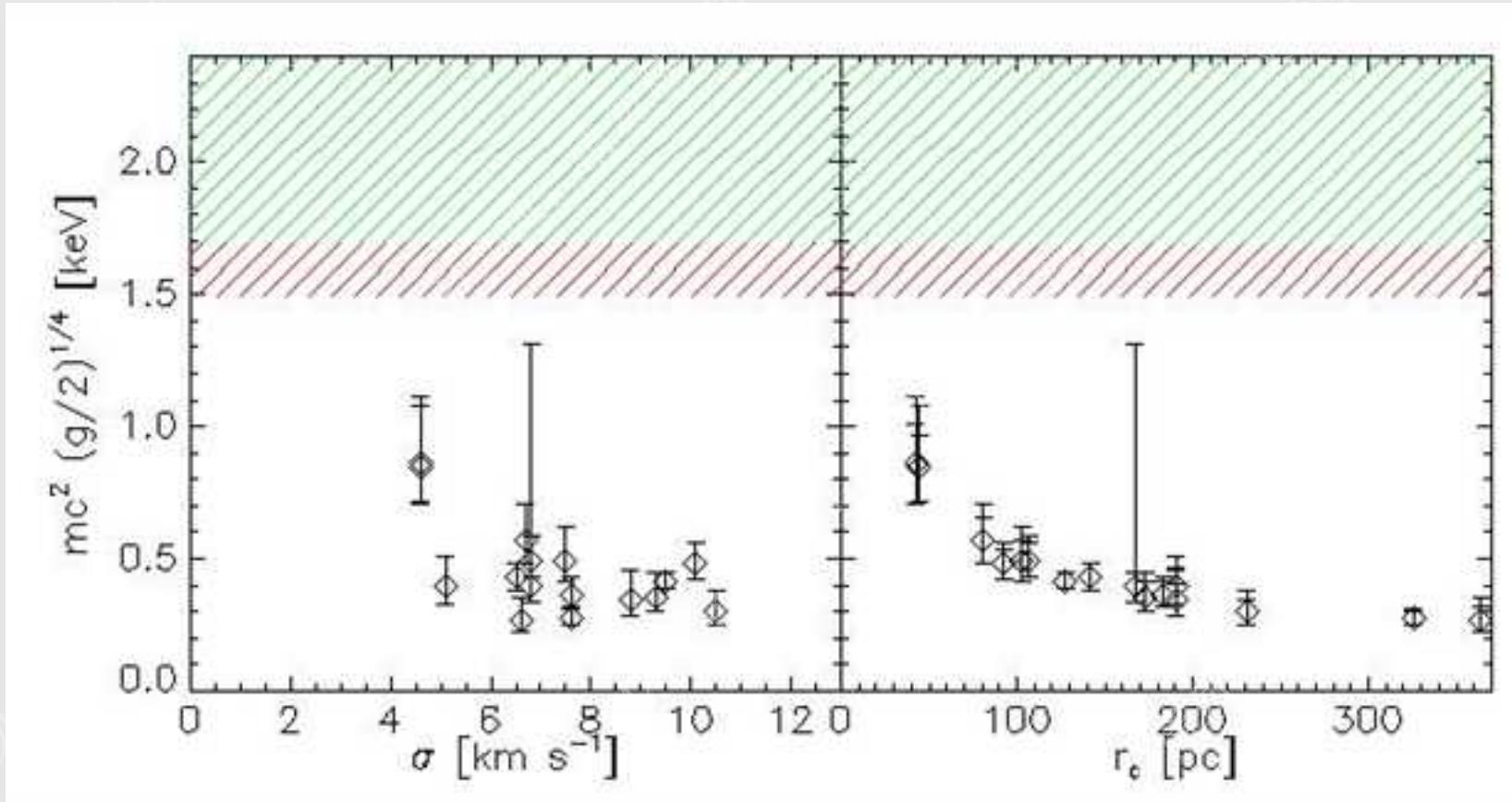
By requiring  $f_0 \leq f_{FD}$ , mass bound for thermal relic Fermi-Dirac particles can be written as

$$m_{FD}^4 \geq \frac{6(2\pi\hbar)^3}{(2\pi)^{5/2} g G \sigma r_h^2}. \quad (13)$$

dSph (1)	$r_h$ [pc] (2)	$\sigma$ [km/s] (3)	$f_0$ [ $M_\odot kpc^{-3} (km/x)^{-3}$ ] (4)	$m_{FD}$ [keV] (5)
dSphs from Gilmore et al. (2007)				
Sextans	$630 \pm 170$	$6.6 \pm 2.3$	$2.69^{+5.06}_{-1.45} \cdot 10^{-6}$	$0.269^{+0.081}_{-0.048}$
Fornax	$400 \pm 103$	$10.5 \pm 2.7$	$4.20^{+6.05}_{-2.09} \cdot 10^{-6}$	$0.301^{+0.075}_{-0.048}$
Leo I	$330 \pm 106$	$8.8 \pm 2.4$	$7.36^{+14.60}_{-4.04} \cdot 10^{-6}$	$0.346^{+0.109}_{-0.063}$
Ursa Minor	$300 \pm 74$	$9.3 \pm 2.8$	$8.42^{+12.81}_{-4.26} \cdot 10^{-6}$	$0.358^{+0.093}_{-0.058}$
Carina	$290 \pm 72$	$6.8 \pm 1.6$	$1.23^{+1.62}_{-0.59} \cdot 10^{-5}$	$0.394^{+0.919}_{-0.059}$
Draco	$221 \pm 16$	$9.5 \pm 1.6$	$1.52^{+0.60}_{-0.39} \cdot 10^{-5}$	$0.415^{+0.036}_{-0.030}$
Bootes	$246 \pm 28$	$6.5^{+2.1}_{-1.3}$	$1.79^{+1.06}_{-0.70} \cdot 10^{-5}$	$0.432^{+0.053}_{-0.050}$
Sculptor	$160 \pm 40$	$10.1 \pm 0.3$	$2.73^{+2.27}_{-1.03} \cdot 10^{-5}$	$0.480^{+0.079}_{-0.054}$
Leo II	$185 \pm 48$	$6.8 \pm 0.7$	$3.03^{+3.13}_{-1.30} \cdot 10^{-5}$	$0.493^{+0.096}_{-0.064}$
dSphs from Simon & Geha (2007)				
Canes Venatici I	$564 \pm 36$	$7.6 \pm 2.2$	$2.92^{+1.77}_{-0.92} \cdot 10^{-6}$	$0.275^{+0.035}_{-0.025}$
Ursa Major I	$318^{+50}_{-39}$	$7.6 \pm 2.4$	$9.17^{+8.24}_{-3.97} \cdot 10^{-6}$	$0.366^{+0.064}_{-0.048}$
Hercules	$330^{+75}_{-52}$	$5.1 \pm 2.4$	$1.27^{+2.11}_{-0.70} \cdot 10^{-5}$	$0.397^{+0.110}_{-0.072}$
Leo T	$178 \pm 39$	$7.5 \pm 2.7$	$2.97^{+4.63}_{-1.50} \cdot 10^{-5}$	$0.490^{+0.131}_{-0.079}$
Ursa Major II	$140 \pm 25$	$6.7 \pm 2.6$	$5.37^{+7.63}_{-2.58} \cdot 10^{-5}$	$0.569^{+0.141}_{-0.086}$
Leo IV	$116^{+26}_{-34}$	$3.3 \pm 2.8$	$1.59^{+19.38}_{-1.01} \cdot 10^{-4}$	$0.746^{+0.676}_{-0.168}$
Coma Berenices I	$77 \pm 10$	$4.6 \pm 2.3$	$2.58^{+4.24}_{-1.23} \cdot 10^{-4}$	$0.843^{+0.233}_{-0.126}$
Canes Venatici II	$74^{+14}_{-10}$	$4.6 \pm 2.4$	$2.80^{+5.02}_{-1.50} \cdot 10^{-4}$	$0.860^{+0.252}_{-0.150}$

Table 1. Parameters for dSphs compiled by Boyarsky et al. (2009)(column 2-3).  $f_0$  is the maximal coarse-grained phase space density bound given by our model (column 4).  $m_{FD}$  is the mass limit for thermal relic warm dark matter particles.

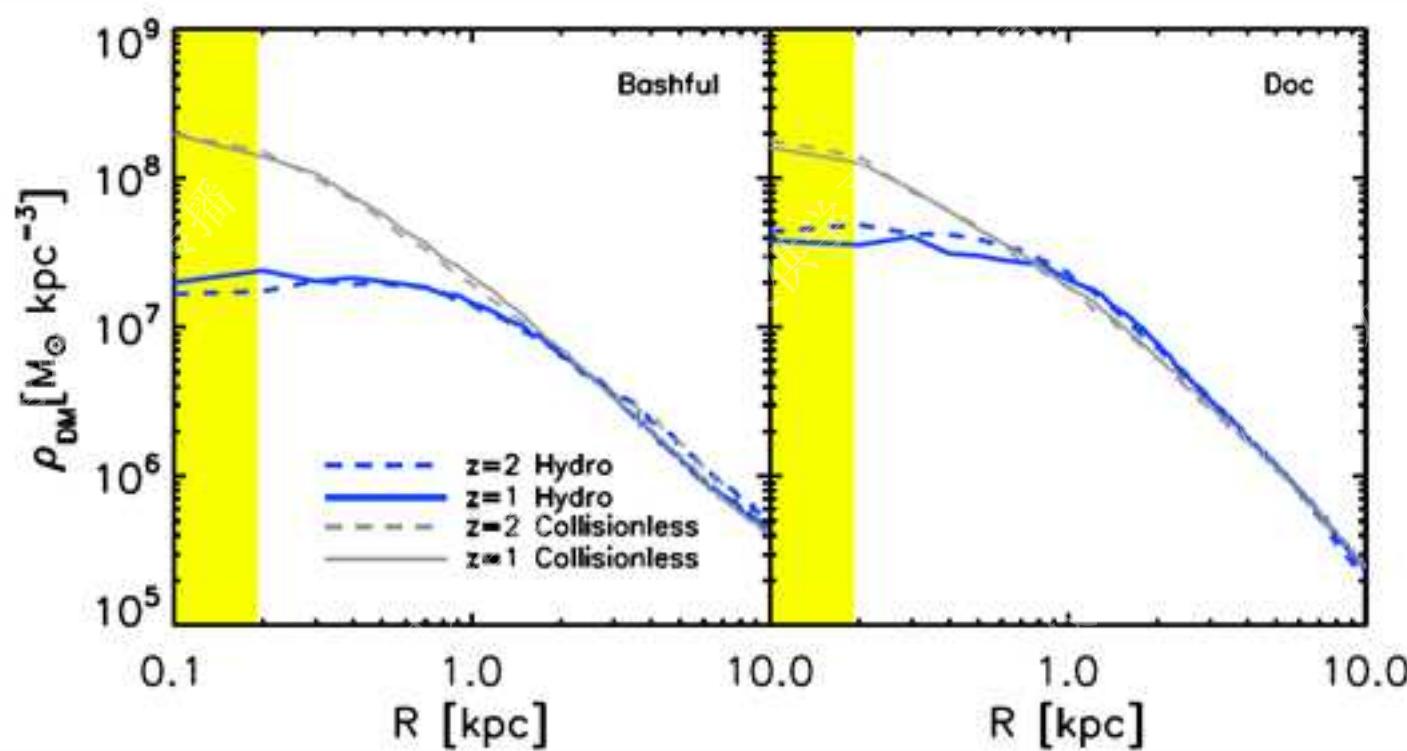
# WDM mass bound



- ~0.5kev,  $M_{\text{cut}} \sim 10^{12} M_{\odot}$ , can not make dwarfs!
  - WDM can NOT account for Dwarf cores Shao et al. 2013

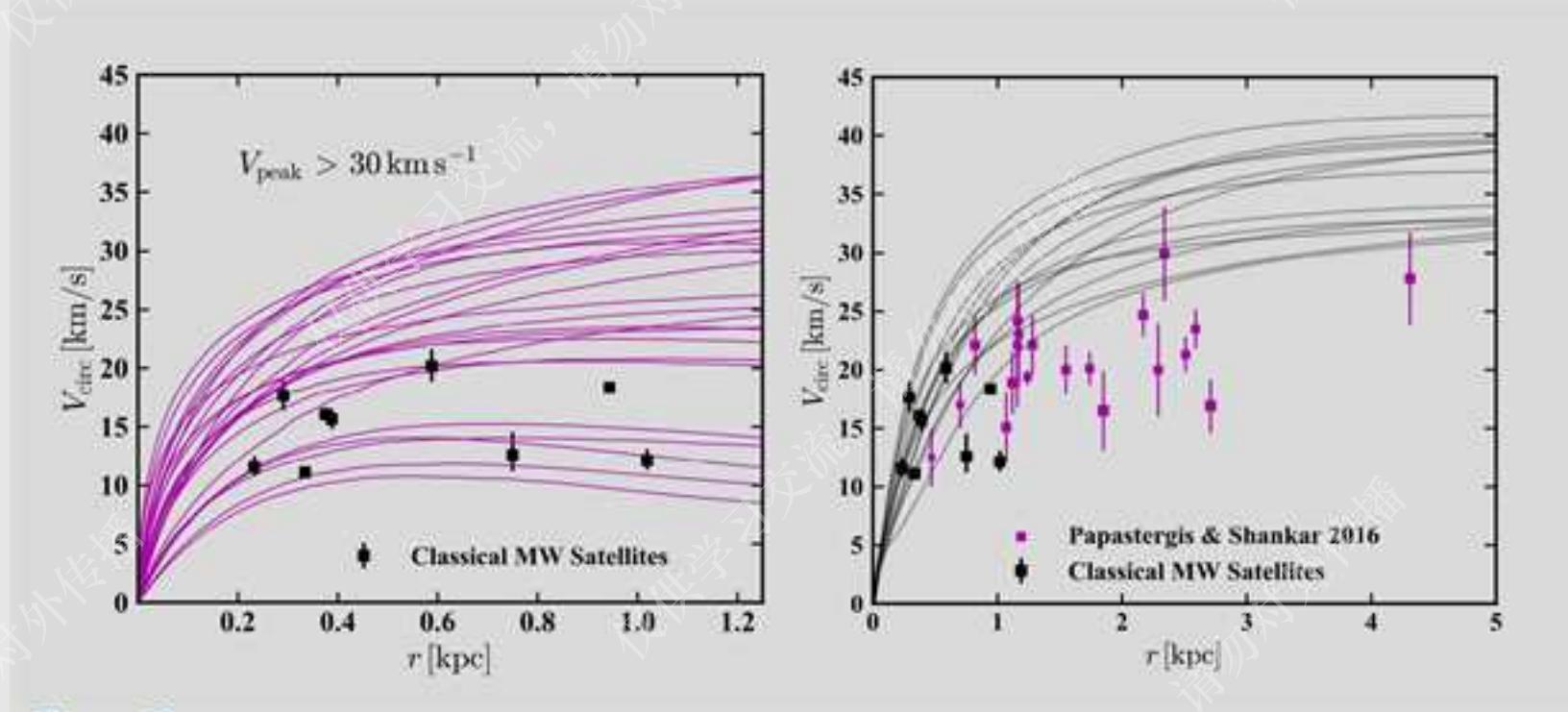
解决方案：超新星爆发反馈加热暗物质粒子，降低中心暗物质密度。

问题：暗的矮星系是否有足够的超新星？

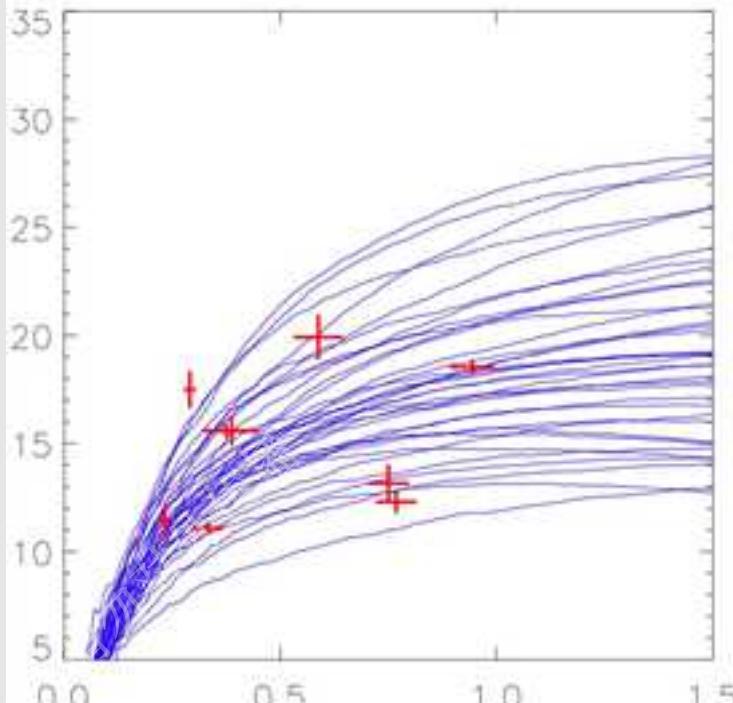
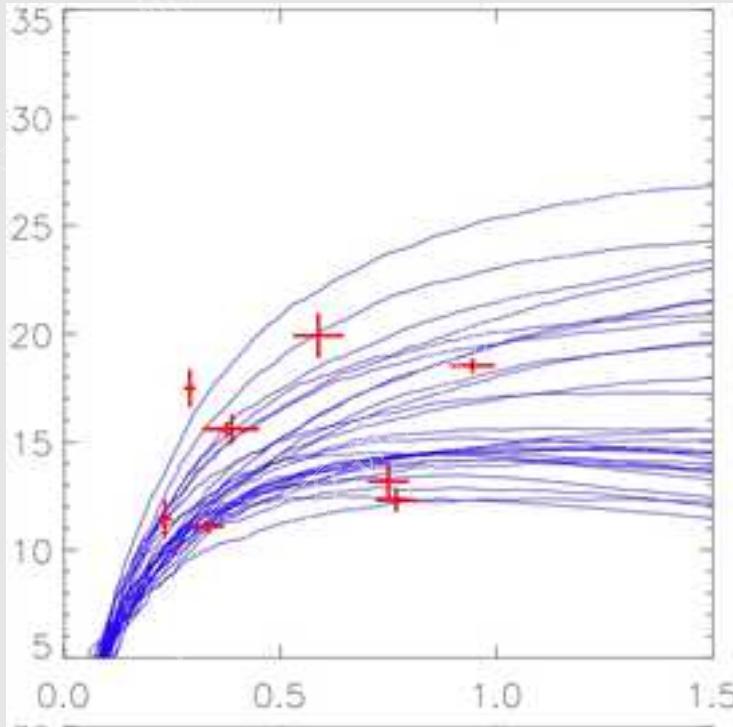


Madau et al. 2014

# Too big to fail problem



$v_c$  [km/s]

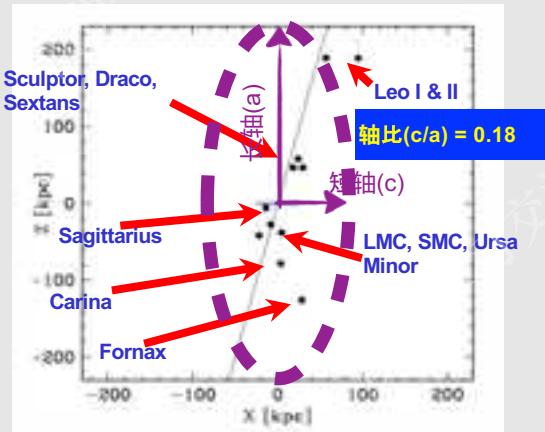


Too big to fail  
Seems not a problem  
In some Hydrodynamic  
simulations

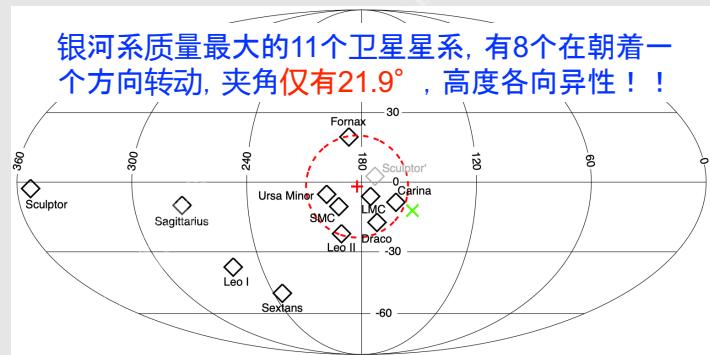
Swala et al. 2016

# 卫星星系盘结构

## 卫星星系空间分布

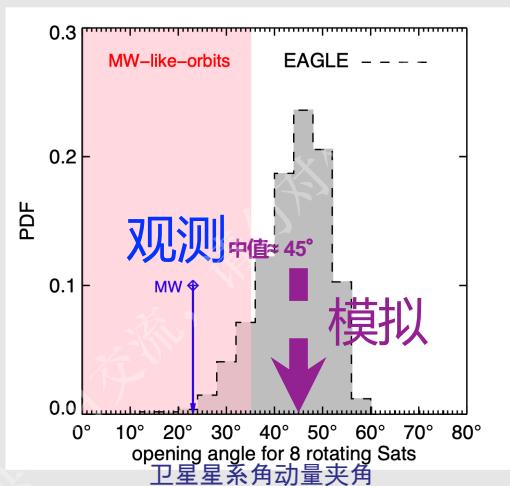
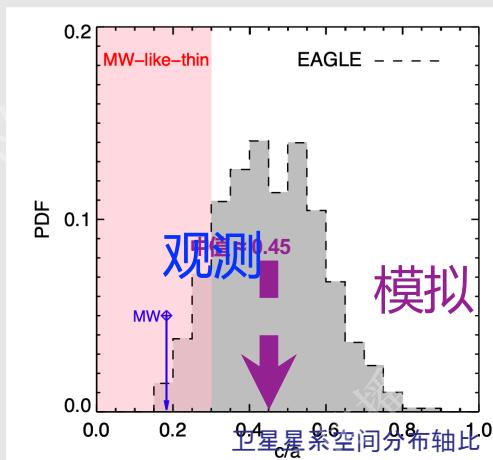


## 卫星星系轨道角动量



Shao ++ 2016 , 2019a

模拟中有 ~ 1% 的  
样本和银河系类  
似！

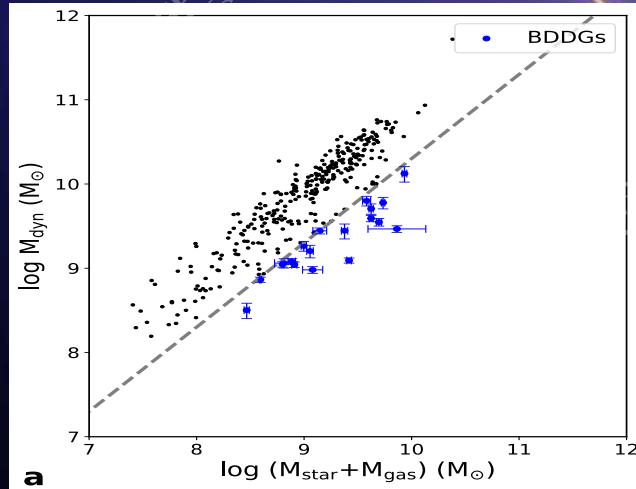


Shao et al. 2020

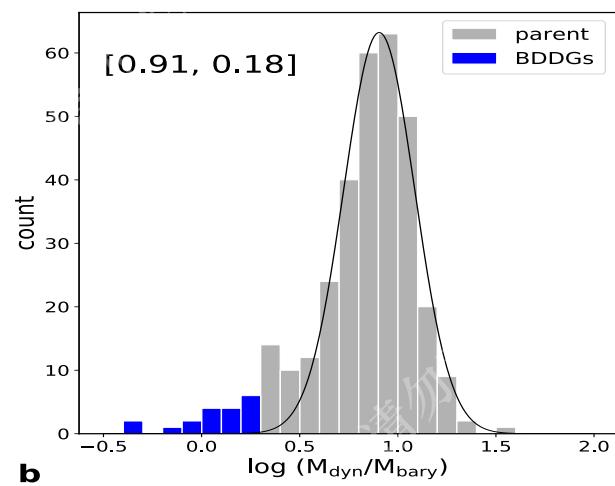
# 暗物质缺失的矮星系

- 利用ALFALFA的HI光谱和SDSS测光估计星系的恒星质量、气体质量和动力学质量。
- 发现324个矮星系中有19个在远大于光学半径的范围内质量由重子物质主导；而在典型的矮星系中，暗物质质量与重子物质质量比在10—1000。

动力学质量



重子物质质量



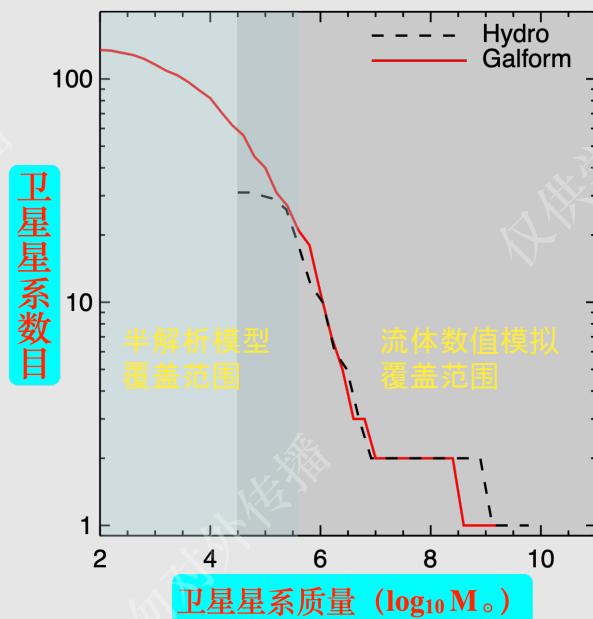
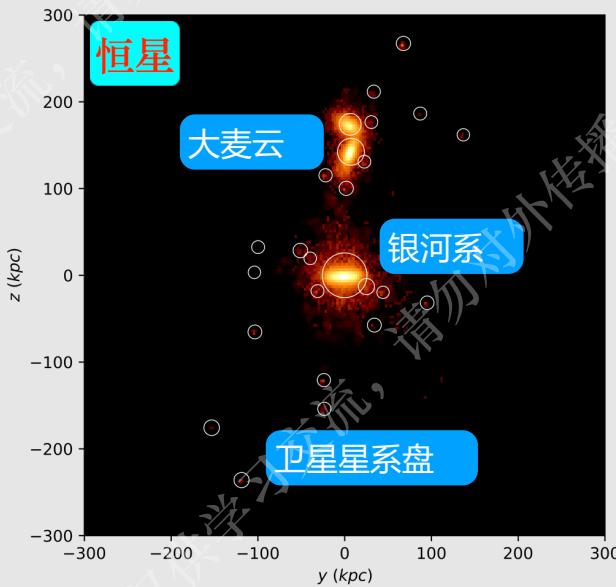
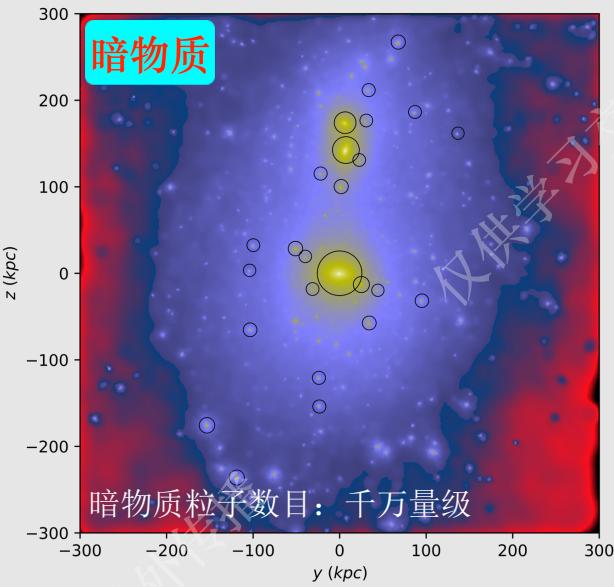
动力学质量/重子物质质量

19个暗物质缺失的矮星系中有14个是孤立星系，不会受到高密度环境，例如潮汐瓦解、星系间相互作用等影响。

Guo et al. 2020



# 喜鹊模拟——银河系高精度数值模拟



- 构建满足多种银河系观测特征的大样本（40）
- 采用全新流体-半解析模型结合的技术，解析最小至 $100 M_\odot$ 的完备卫星星系样本

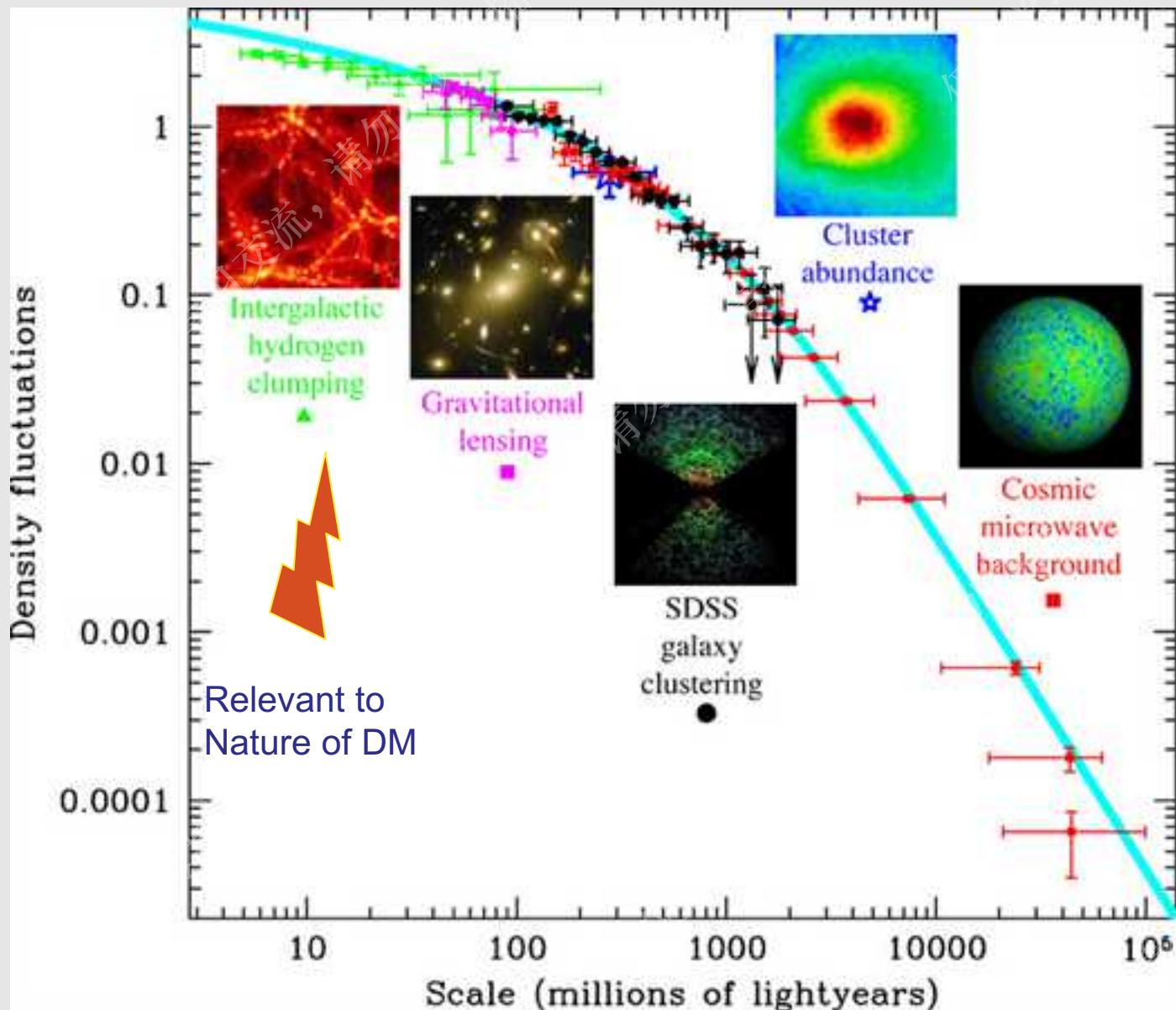
# 暗物质不一定非得是 WIMP 冷暗物质未必是对的

Wimps, Axions (冷暗物质)

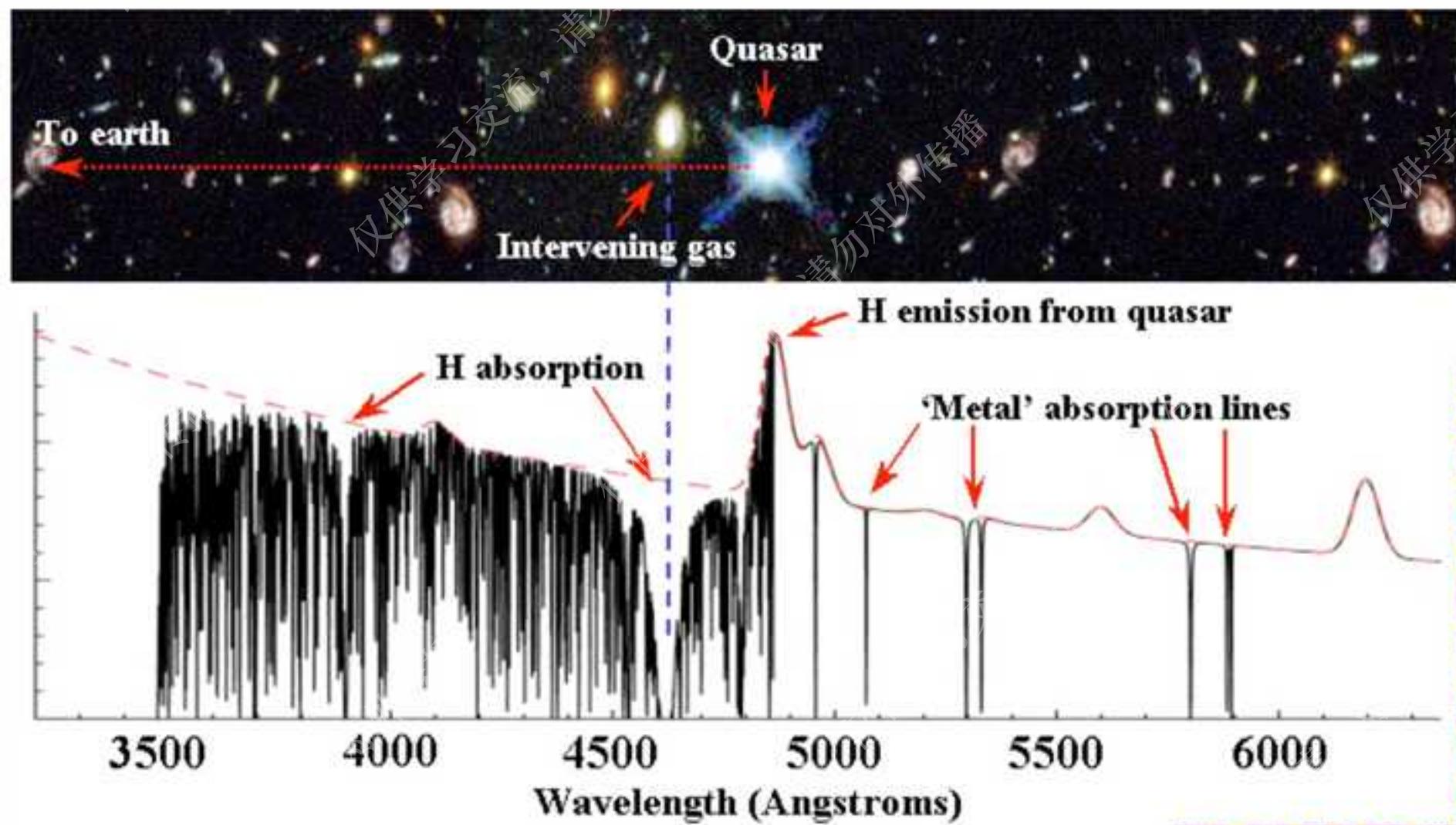
Sterile neutrino , gravtino (温暗物质)

Self-interactive dark matter

# Matter Power Spectrum constraints from observations



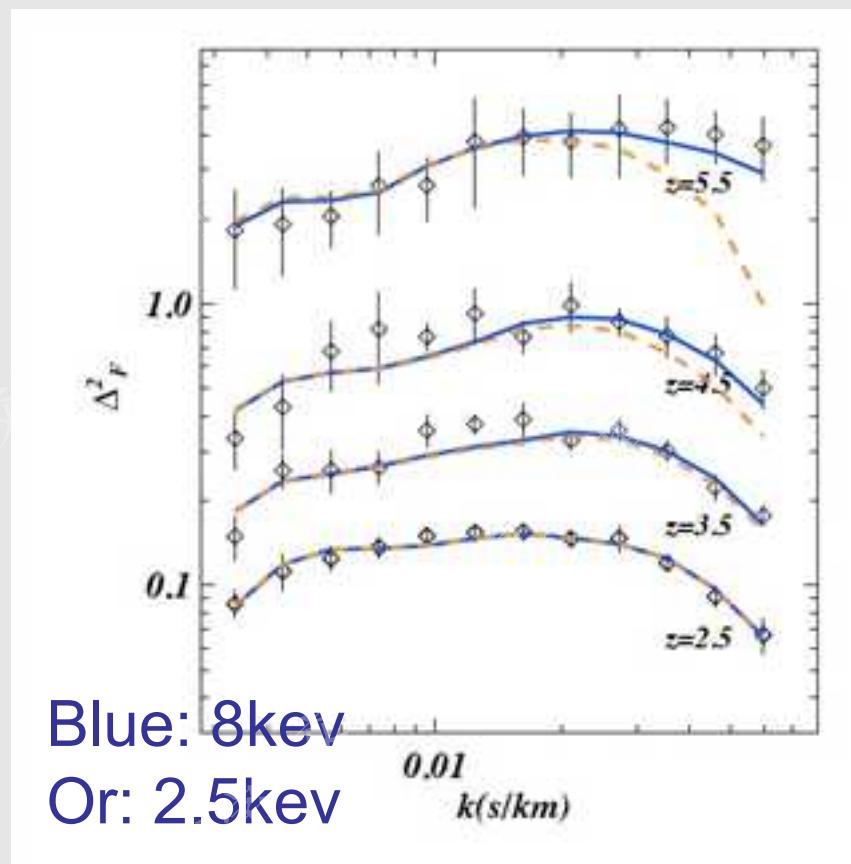
# Lya ps constraints on WDM mass



# Ly $\alpha$ ps constrains on WDM mass

Difference in Flux PS mostly seen on high  $z$ , at which both data and simulation are more uncertain.

Simulation boxsize  
Different Hydro-dynamical solver



Viel et al. 2008

# 探测超对称暗物质粒子手段

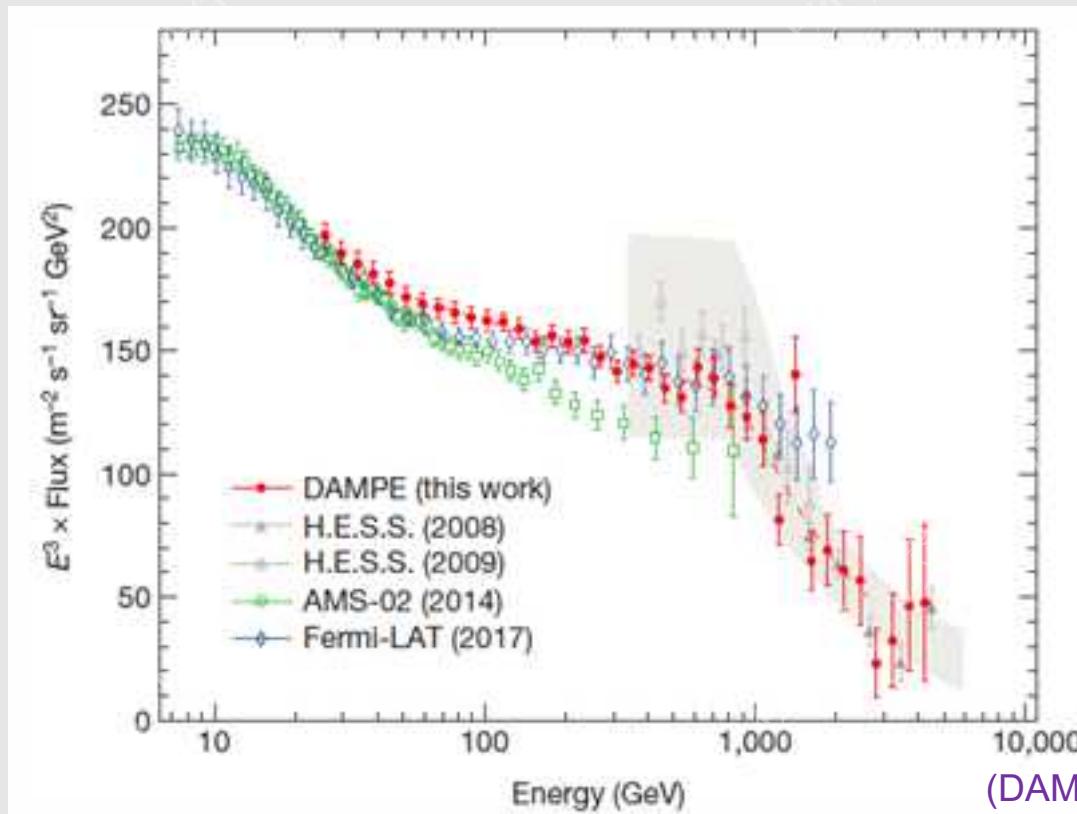
- 非加速器物理
  - 直接探测方法

测量暗物质粒子与普通粒子之间极其罕见的散射事例
  - 间接探测方法

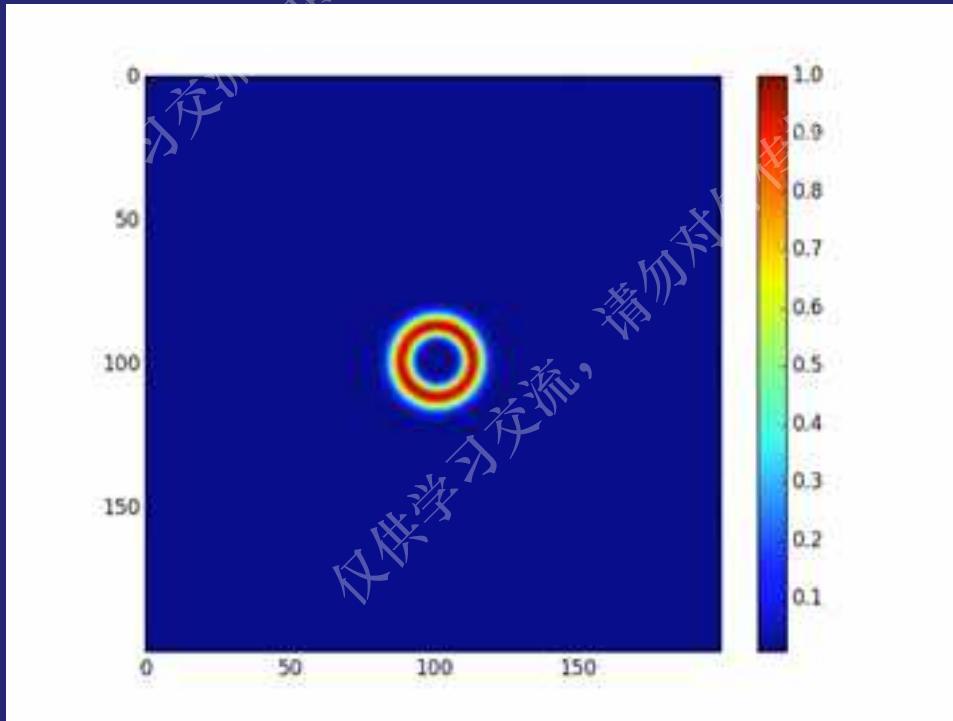
测量暗物质粒子湮灭所产生的次级粒子  $\gamma$ 光子,  $e^+ e^-$ , 中微子
- 目前国际上已有大量暗物质实验在进行, 基本上是非加速器实验。相对于加速器实验, 非加速器实验造价十分低廉
- 加速器物理: 欧洲核子中心的LHC (大型强子对撞机)

# DAMPE results

- Cosmic ray ( $e^-$  and  $e^+$ ) energy power spectrum
- A tentative 1.4 TeV peak



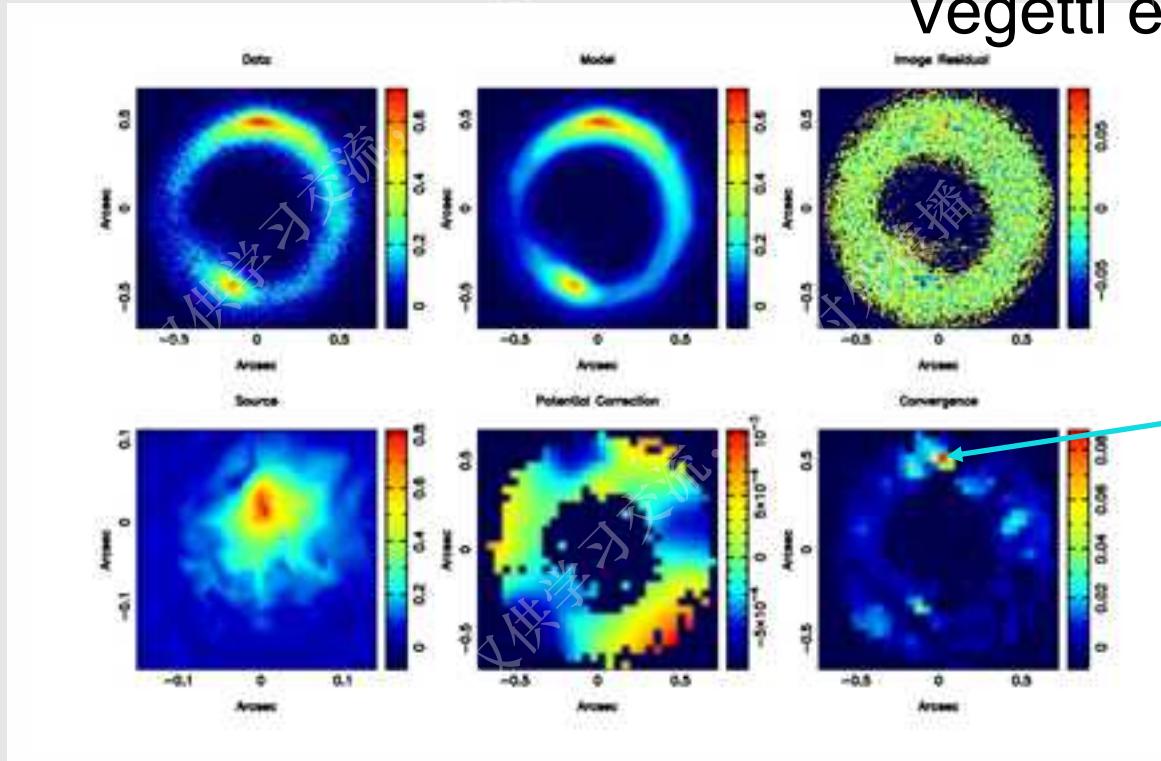
# 子暗晕可以扰动爱因斯坦环



The perturbation on a Einstein ring with subhaloes at different position. The cleanest way to detect very low mass subhalo.

# 通过测量Einstein Ring扰动测量子结构比例来限制暗物质性质

Vegetti et al. 2012

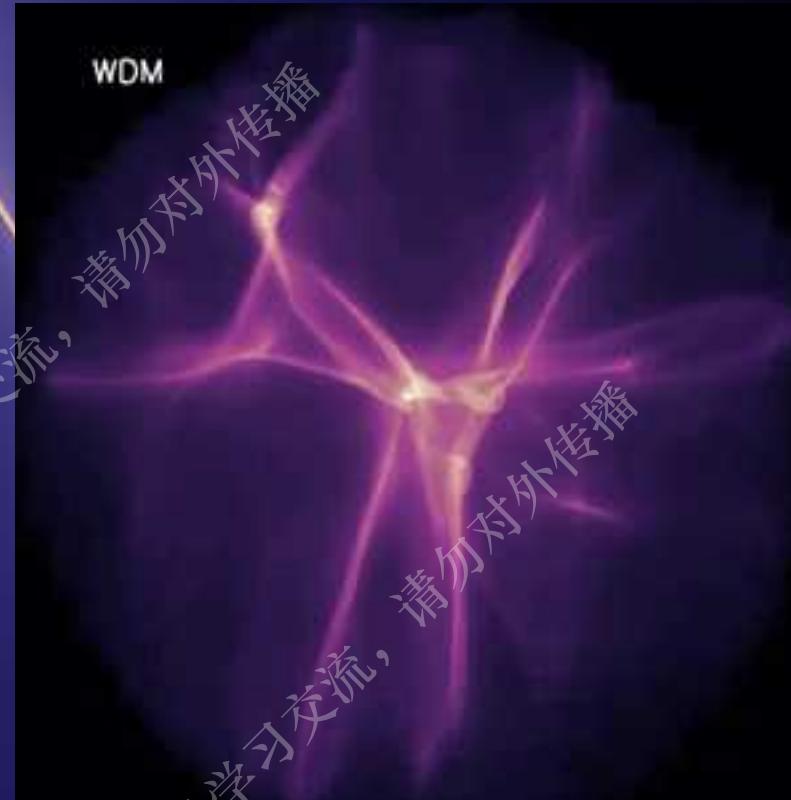
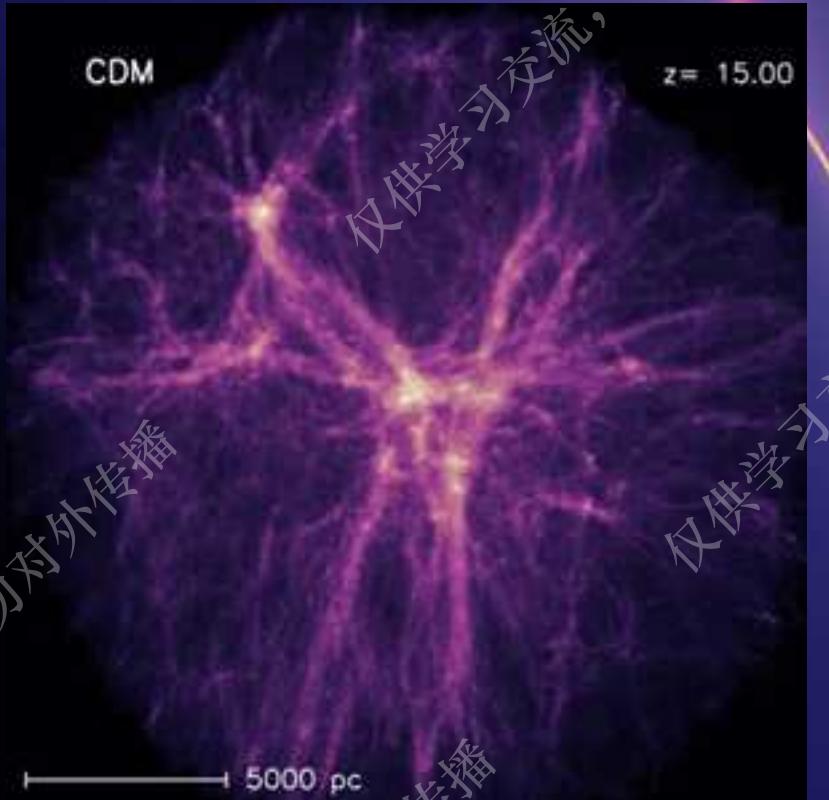


subhalo of  
1e8 Msun

1. model the system with smooth density(host halo mass distribution)
2. model the residual(subhalo)
3. calculate the significance of subhalo detection

# Are filaments special places to look at?

Filaments in WDM are more smooth



$M_{\text{dm}} = 3 \text{ kev}$ ,  $M_{\text{fs}} \sim 3 \times 10^8 \text{ solar mases}$

冷暗物质

# 第一代恒星

- 经典理论下，恒星只能在暗晕中形成
- 温暗物质纤维结构可以产生恒星



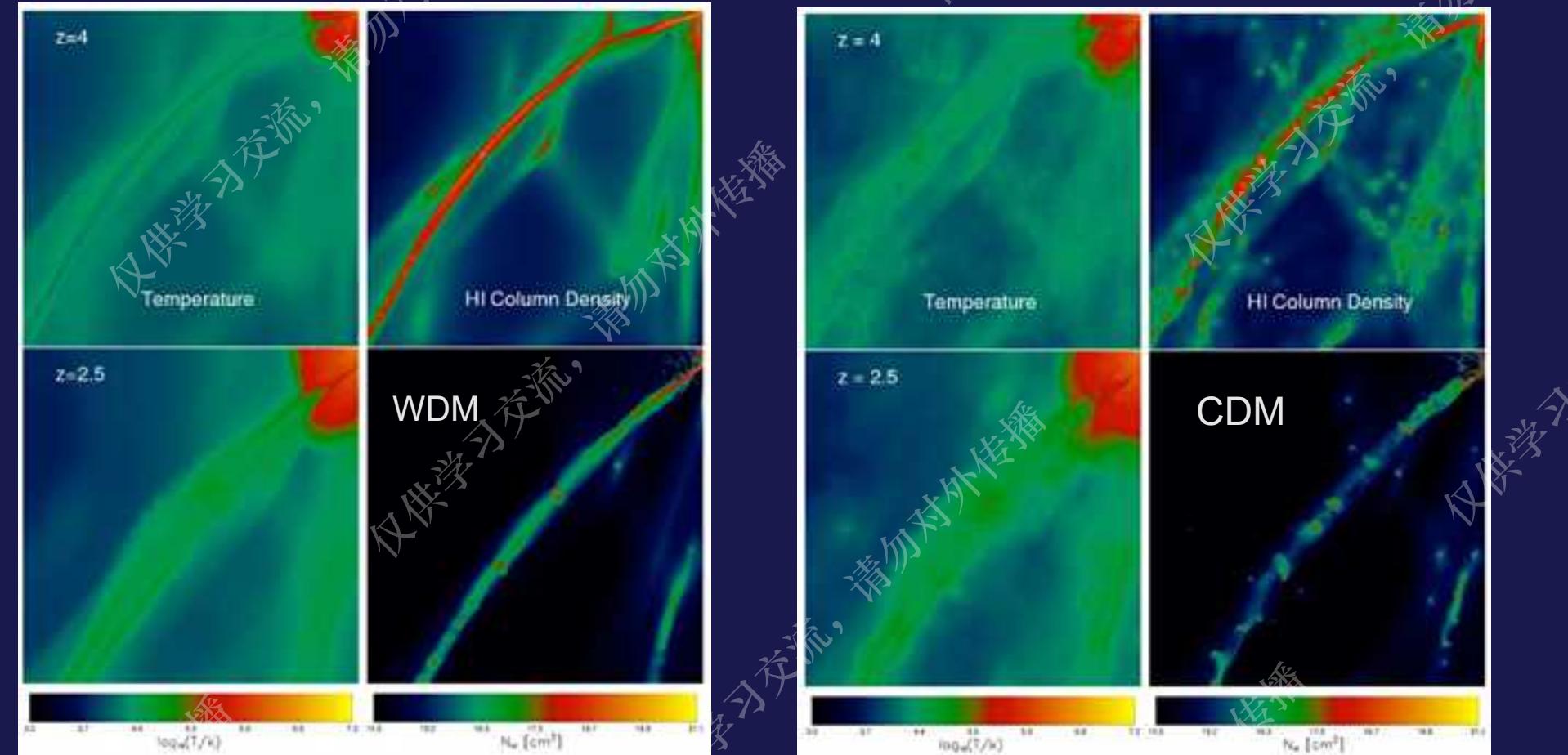
Gao & Theuns, 2007 Science

星爆方式发生，极高亮度

CDM

WDM

温暗物质



Can we see them ? Ly $\alpha$  emission, HI

# 总结

- 纯暗物质物理体系下，暗晕结构有明确的结果。但或许和一些观测有冲突
- 当考虑重子物质, well-known小尺度问题似乎都可以解决
- 小尺度观测结果也存在一些争议
- 标准宇宙学小尺度挑战不足以引入修改暗物质模型
- 暗物质属性天文学研究依然十分重要